



Knowledge Organisers

Year 9

CORE

GOOD TO KNOW...

HOW TO....

- The priority of operations is: brackets, indices, division, multiplication, addition and subtraction. This is called BIDMAS
 - When there is only addition and subtraction or multiplication and division on the same line, we work from left to right.
- Finding the square root is the inverse of finding the square
- Finding the cube root is the inverse of finding the cube
- To round a number to 1 decimal place (1 d.p.), look at the digit in the second decimal place. If it is 5 or more, round up.
- To multiply decimals, ignore the decimal and work out the normal calculation, then put the number of total digits after the decimal place in the question back into the answer. I.e. $1.1 \times 1.2 = 1.32$
- To divide by a decimal, multiply both numbers by a power of ten (10, 100, 1000 etc.) until you have a whole number to divide by. Then work out the division using the bus stop method.
- A factor is a number that goes into another number without leaving a remainder i.e. 5 is a factor of 20 as 5 goes into 20 four times.
- A multiple is the times tables of a number i.e. the first three multiples of 6 are 6, 12 and 18.



Round 68.1572 to the nearest:

Whole number: 68 2 decimal places: 68.16

1 decimal place: 68.2 3 decimal places:

Round 7.82438 to 3 significant figures	→	<u>7.82</u>
Round 4537 to 1 significant figure	→	<u>5 000</u>
Round 37.85672 to 3 significant figures	→	<u>37.9</u>
Round 6973 to 2 significant figures	→	<u>7000</u>

Laws of indices

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{m \times n}$$

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

Factors of 20: 1, 2, 4, 5, 10, 20

HCF of 30 and 20: 10

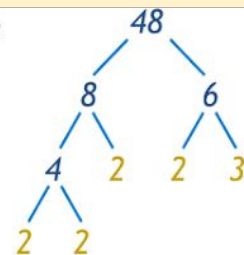
Find the Least Common Multiple

8, 4, 6

8 → 8, 16, 24, 32, 40, 48

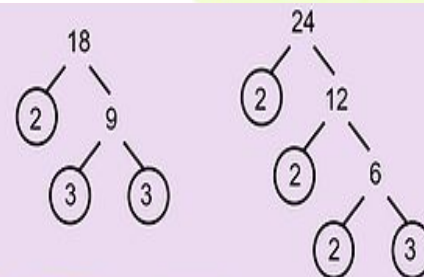
4 → 4, 8, 12, 16, 20, 24, 28, 32

6 → 6, 12, 18, 24, 30, 36



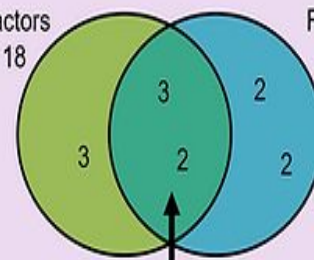
$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

Calculate the HCF and LCM of 18 and 24.



Factors of 18

Factors of 24



Factors of both

$$\text{HCF} = 2 \times 3 = 6$$

$$\text{LCM} = 3 \times 2 \times 3 \times 2 \times 2 = 72$$

CORE

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- A factor is a number that goes into another number without leaving a remainder i.e. 5 is a factor of 20 as 5 goes into 20 four times.
- A multiple is the times tables of a number i.e. the first three multiples of 6 are 6, 12 and 18.

Indices

- $2 \times 2 \times 2 \times 2$ can be written 2^4

Standard Form

$$a \times 10^n$$

Where $1 \leq a < 10$ and n is an integer.

If n is positive, multiply 'a' by 10^n times.

If n is negative, divide 'a' by 10^n times (this will decrease the value and be a decimal).

GOOD TO KNOW...

- When there are m ways of doing one task and n ways of doing another, the total number of ways of doing the first task and then the second task is $m \times n$ ways.
- A factorial is the result of multiplying a sequence of descending integers. I.e. $4! = 4 \times 3 \times 2 \times 1 = 24$

Negative Exponents

$$a^{-n} = \frac{1}{a^n} \quad \text{For } a \neq 0$$

a^{-n} is a reciprocal of a^n

Example:

$$3^{-2} = \frac{1}{3^2}$$

$$\left(\frac{2}{5}\right)^{-6} = \left(\frac{5}{2}\right)^6$$

Fractional Indices

Numerator - Power

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$$

Denominator - Root

Examples:

$$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$$

$$25^{\frac{3}{2}} = \left(\sqrt{25}\right)^3 = 5^3 = 125$$

Laws of indices

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{m \times n}$$

HOW TO....

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

Factors of 20: 1, 2, 4, 5, 10, 20

HCF of 30 and 20: 10

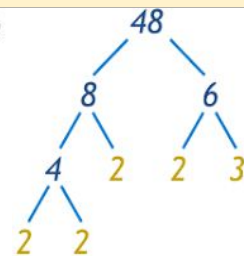
Find the Least Common Multiple

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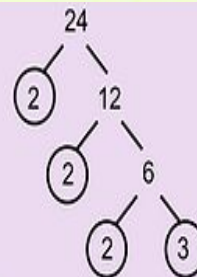
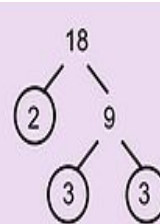
4 → 4, 8, 12, 16, 20, 24, 28, 32

6 → 6, 12, 18, 24, 30, 36



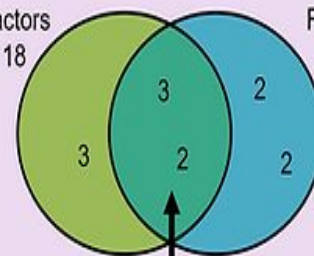
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Calculate the HCF and LCM of 18 and 24.



Factors of 18

Factors of 24



Factors of both

$$\text{HCF} = 2 \times 3 = 6$$

$$\text{LCM} = 3 \times 2 \times 3 \times 2 \times 2 = 72$$

CORE

- A term is a number, letter, or a number and a letter multiplied together i.e. x , $3a$, $7y^2$ are all terms
- 'Like terms' contain the same letter to the same power (or contain no letters at all). You can simplify expressions by collecting like terms. i.e. $2x + 3x = 5x$
- Terms can be simplified when multiplying or dividing even when they are not like terms. i.e. $a \times b = ab$
 - When multiplying, write the letters in alphabetical order
 - Write the number before the letter(s)
- Substitution means putting numbers in place of letters.
- The factors of a term are all of the numbers and letters that divide exactly into it.
- A common factor is a factor of two or more terms.
- Expand - multiply term outside the bracket by all terms inside the brackets to eliminate brackets

Laws of indices

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{m \times n}$$

GOOD TO KNOW...

Terms can be simplified when multiplying or dividing, even when they are not like terms.

$$a \times b = ab \quad x \div y = \frac{x}{y}$$

When multiplying:

- write letters in alphabetical order
- write numbers before letters

Evaluate $3a - 2b$, for $a = 10$ and $b = 4$

$$\begin{aligned} 3a - 2b & \quad (a = 10 \quad b = 4) \\ &= 3(10) - 2(4) \\ &= 30 - 8 \\ &= 22 \quad \checkmark \end{aligned}$$

Like Terms	Unlike Terms	Factorise	Answer
$2x + 19x$	$2x + 19a$	$7x + 14$	$7(x + 2)$
$4w - 10w$	$4w - 10w^2$	$45 - 27k$	$9(5 - 3k)$
$14.2r - 12r$	$12r - 12s$	$12ab + 7b$	$b(12a + 7)$
$32a^2 + 9a^2$	$32a^2 + 9a^3$	$y^2 - 9y$	$y(y - 9)$
$8y + 5y$	$8y + 5$	$8t - 32t^2$	$8t(1 - 4t)$

HOW TO....

Collecting Like Terms

Ex1 $x + 4y + 6x + 2y = 7x + 6y$

Ex $3x + y - 2x + 4y = x + 5y$

$3(a+4) = 3a + 12$

$4(a-5) = 4a - 20$

Factorising

$3x + 6 = 3(x + 2)$

Expanding brackets

Expand & Simplify...

$5(x + 3) + 6(x - 4)$

$5x + 15 + 6x - 24$
 $11x - 9$

1) $3a + 6y = 3(a + 2y)$

$4x + 32 = 4(x + 8)$

CORE

GOOD TO KNOW...

HOW TO....

Indices

- $2 \times 2 \times 2 \times 2$ can be written 2^4

- When multiplying powers add the powers
e.g. $6^4 \times 6^7 = 6^{11}$ OR $a^3 \times a^5 = a^8$
- When dividing powers subtract the powers
e.g. $6^8 \div 6^5 = 6^3$ OR $a^9 \div a^5 = a^4$
- When in brackets multiply the powers
e.g. $(8^4)^3 = 8^{12}$ OR $(x^5)^2 = x^{10}$
- Any number to the power of zero is 1
- Expand - multiply term outside the bracket by all terms inside the brackets to eliminate brackets
- The factors of a term are all of the numbers and letters that divide exactly into it.
- A common factor is a factor of two or more terms.
- The subject of a formula is the letter on its own, on one side of the equals sign.
- A term is a number, letter, or a number and a letter multiplied together i.e. x , $3a$, $7y^2$ are all terms
- An expression contains letter and/ or number terms but no equal sign.
- An equation has an equals sign, letter terms and numbers. You can solve it to find the value of the letter.
- An identity is true for all values of letters
- A formula has an equals sign and letters to represent different quantities. The letters are variables as their values can vary.

Q1) Expand: $(x + 3)(x - 2)$

	x	-2
x	x^2	$-2x$
$+3$	$+3x$	-6

$$(x + 3)(x - 2) = x^2 + x - 6$$

Factorise	Answer
$7x + 14$	$7(x + 2)$
$45 - 27k$	$9(5 - 3k)$
$12ab + 7b$	$b(12a + 7)$
$y^2 - 9y$	$y(y - 9)$
$8t - 32t^2$	$8t(1 - 4t)$
$16gh + 28gf$	$4g(4h + 7f)$
$21w^2z - 77wx$	$7w(3wz - 11x)$

Expand & Simplify...

$$5(x + 3) + 6(x - 4)$$

$$5x + 15 + 6x - 24$$

$$11x - 9$$

Finding nth term of linear sequence

1) 6, 10, 14, 18, 22 **The sequence increases by 4, so the nth term starts with 4n**

Now compare the sequence to the 4 times table

6, 10, 14, 18, 22	Each term is 2 bigger than the 4 times table
↑+2 ↑+2 ↑+2 ↑+2	
4, 8, 12, 16, 20	So the nth term is $4n + 2$

a Make a the subject of the formula $v^2 = u^2 + 2as$

b Make x the subject of the formula $y = \frac{ax + b}{c}$

a $v^2 = u^2 + 2as$

b $y = \frac{ax + b}{c}$

$v^2 - u^2 = 2as$ — Subtract u^2 from both sides.

$cy = ax + b$ — Multiply both sides by c .

$\frac{v^2 - u^2}{2s} = a$ — Divide both sides by $2s$.

$cy - b = ax$ — Subtract b from both sides.

$a = \frac{v^2 - u^2}{2s}$ — Re-write in the form $a = \dots$

$\frac{cy - b}{a} = x$ — Divide both sides by a .

$x = \frac{cy - b}{a}$ — Re-write in the form $x = \dots$

CORE

Graphs, tables and charts are used to display, interpret and compare data.

Discrete Data - Can only have particular values, e.g shoe size.

Continuous Data - Measured and can have any values e.g length and time.

Grouped Frequency Table - Contains sorted data in groups called classes.

Two-way Table - Divides data into groups in rows across and in columns down the table. You can calculate totals across and down.

Stem and Leaf Diagram - Shows numerical data split into "leaves" (usually the last digit) and a "stem" (the other digits).

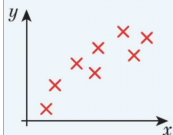
Pie Chart - A circle divided into sectors, each sector represents a set of data.

Scatter Graphs - Shows the relationship between two sets of data. Plot the points with crosses. Do not join them up.

- **Correlation:** Relationship between the sets of data.
- **Outlier:** A value that does not fit the pattern.
- **Line of best fit:** A straight line drawn through the middle of the points representing the trend.

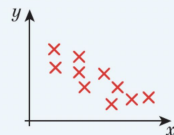
GOOD TO KNOW...

Positive correlation



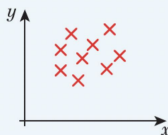
As x increases
 y increases

Negative correlation



As x increases
 y decreases

No correlation



No relationship
between x and y

	Baseball	Basketball	Football	Total
Male	13	15	20	48
Female	23	16	13	52
Total	36	31	33	100

11, 4, 27, 18, 18, 3, 24, 22, 11, 22, 18, 11, 18, 7, 29, 18, 11, 6, 29, 11

Intervals	Tally Marks	Frequency
0 - 5		2
5 - 10		2
10 - 15		5
15 - 20		5
20 - 25		3
25 - 30		3

Here,

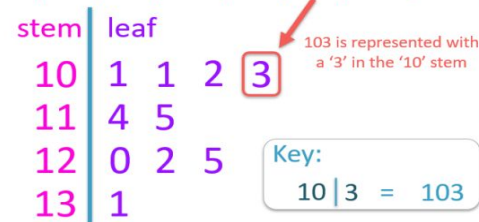
0-5, 5-10, 10-15, are class Intervals

HOW TO....

Only place the last digit of each number in the 'leaf'

Arrange the numbers from smallest to largest

101, 131, 114, 102, 125, 101, 115, 103, 120, 122



Place all other digits of the number in the 'stem'

The table shows the match results of a football team. Draw a pie chart to represent the data.

Result	Won	Drawn	Lost
Frequency	28	12	20

Total number of games = $28 + 12 + 20 = 60$

$$\begin{aligned} &\div 60 \quad 60 \text{ games} : 360^\circ \quad \div 60 \\ &\quad \quad \quad 1 \text{ game} : 6^\circ \end{aligned}$$

The total number of games is the total frequency.

$$1 \text{ game} = 360 \div 60 = 6^\circ$$

Work out the angle for one game.

$$\text{Won: } 28 \times 6^\circ = 168^\circ$$

Work out the angle for each result.

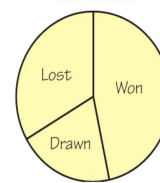
$$\text{Drawn: } 12 \times 6^\circ = 72^\circ$$

$$\text{Lost: } 20 \times 6^\circ = 120^\circ$$

Check that your angles total 360° .

$$\text{Check: } 168 + 72 + 120 = 360$$

Team results



Draw the pie chart. Give it a title and label each section, or make a key.

Year 9 Term 3 Maths Knowledge Organiser [H unit 3-Interpreting&representing data] TKAT

CORE

GOOD TO KNOW...

HOW TO....

Stem and Leaf Diagram - Shows numerical data split into "leaves" (usually the last digit) and a "stem" (the other digits).

Frequency Polygon - To draw a frequency polygon, plot the frequency against the midpoints for each group.

Time-series Graphs - A time-series graph is a line graph with time plotted on the horizontal axis.

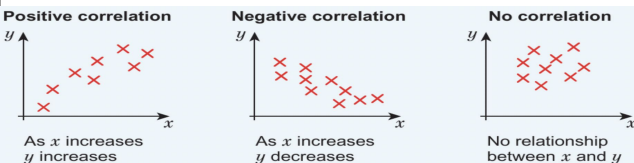
Scatter Graphs - Shows the relationship between two sets of data. Plot the points with crosses. Do not join them up.

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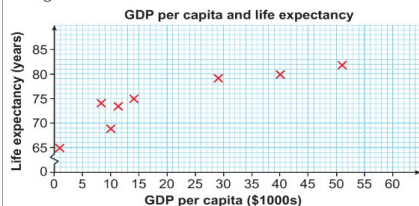
Averages and Range

The modal class has the highest frequency. Make sure to write down the class and not the frequency.

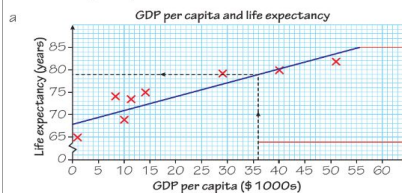
The median is where the middle data point lies.



The scatter graph shows the GDP per capita (in \$1000s) and life expectancy (in years) for eight countries.



- Draw a line of best fit.
- The GDP per capita in the UK is \$36,000. Estimate the life expectancy of a baby born in the UK.



- Estimated life expectancy in the UK is 79 years.

Position a transparent ruler over your scatter graph so it follows the overall trend. Move it slightly so you have roughly the same number of points above and below the line.

To estimate life expectancy, start at \$36,000 on the horizontal axis, go up to the line of best fit and read off the answer on the vertical axis.

The annual salaries of employees working in an ICT company are displayed in the back-to-back stem and leaf diagram.

Key	Male	Female							
8 1	represents a salary of £18,000	1 9	represents a salary of £19,000						
	Male	Female							
	8	1	9	9					
	9	5	2	0	2	1	2	6	7
	8	7	3	0	3	0	4	4	
					4	5	6		
					5	4	8		

Compare the distribution of salaries of the male and female employees.

Male range: $38,000 - 18,000 = £20,000$

Female range: $58,000 - 19,000 = £39,000$

There are 9 males, so median male salary is: $\frac{9+1}{2} = 5$ th value = £29,000

There are 13 females so median female salary is: $\frac{13+1}{2} = 7$ th value = £30,000

Female employees' salaries have a larger range but the median salaries of men and women are similar.

Write a sentence comparing ranges and medians.

The table shows the times, T , taken for 100 people to queue for a rollercoaster at a theme park

- Estimate the mean waiting time.
- Explain why the mean is only an estimate.

The third column gives an estimate of the waiting time in each class.

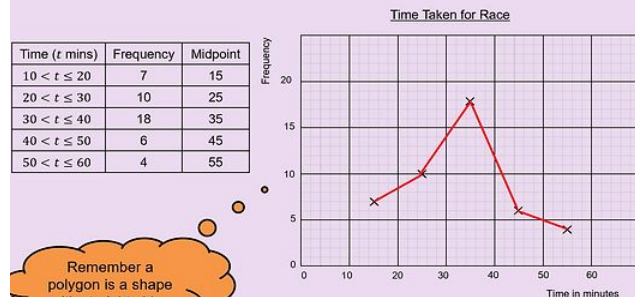
Time, T (mins)	Frequency, f	Class midpoint, x	xf
$0 \leq T < 20$	14	10	$10 \times 14 = 140$
$20 \leq T < 40$	55	30	$30 \times 55 = 1650$
$40 \leq T < 60$	31	50	$50 \times 31 = 1550$
Total	100		3340

The fourth column gives an estimate of the total waiting time in each class.

$$\text{Mean} = \frac{\text{sum of waiting times}}{\text{total number of people}} = \frac{3340}{100} = 33.4 \text{ minutes}$$

- The mean is an estimate because we don't know the exact times taken.

A frequency polygon can be drawn directly from the frequency table by using by finding the midpoint of each class interval.



Remember a polygon is a shape with straight sides

CORE	GOOD TO KNOW...	HOW TO....
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Operations with Fractions

- **Add/ subtract** fractions by finding equivalent fractions with the same denominator
- **Multiply** fractions by multiplying the numerators together and the denominators together
- **Divide** fractions by following the KFC rule: keep the first fraction as it is, flip the second fraction around so the numerator becomes the denominator and change the sign from a divide to times.

Finding Percentages

- 50% - Divide amount by 2
- 10% - Divide amount by 10
- 1% - Divide amount by 100

Keywords

Fraction - A fraction represents a part of a whole.

Decimal - A number with a decimal point in it.

Percentage - A part of a whole expressed in hundredths.
e.g 1% of £100 = £1

Numerator - The part of a fraction that is above the line and signifies the number to be divided by the denominator.

Denominator - The part of a fraction that is below the line and that functions as the divisor of the numerator.

Simple interest - is the interest calculated only on the original amount invested. It is the same each year.

GOOD TO KNOW...

Mixed Number - A number consisting of a whole number and a proper fraction.

Improper Fraction - A fraction whose numerator is larger than the denominator.

Find the simple interest when £5000 is invested at 2.75% per annum over 2 years.

2.75% = 0.0275 ———— Convert the percentage to a decimal multiplier.

5000 × 0.0275 = £137.50 ———— This is the interest earned over 1 year.

£137.50 × 2 = £275 ———— Multiply your answer by 2.

Communication hint
Per annum or p.a. means 'each year'.

There are 20 students in a class. 6 are male. What percentage of the class is male?

Method A: $\frac{6}{20} \times 100\% = 6 \times \frac{100}{20}\%$
= 30%

Method B: $\frac{6}{20} = \frac{30}{100} = 30\%$

Convert to a fraction with denominator 100.

Work out

$$\frac{3}{4} \times \frac{2}{7}$$

$$\frac{3 \times 2}{4 \times 7} = \frac{6}{28} = \frac{3}{14}$$

Work out

$$\frac{3}{4} \div \frac{2}{7}$$

$$\frac{3}{4} \times \frac{7}{2} = \frac{21}{8} = 2 \frac{5}{8}$$

Work out

$$\frac{3}{4} + \frac{2}{7}$$

$$= \frac{21}{28} + \frac{8}{28}$$

$$= \frac{29}{28}$$

$$= 1 \frac{1}{28}$$

Work out

$$\frac{3}{4} - \frac{2}{7}$$

$$= \frac{21}{28} - \frac{8}{28}$$

$$= \frac{13}{28}$$

HOW TO....

$$2\frac{3}{4} = \frac{(4 \times 2) + 3}{4} = \frac{8 + 3}{4} = \frac{11}{4}$$

Mixed Number

Improper Fraction

Write $\frac{7}{8}$ as a decimal.

$$\frac{7}{8} = 8 \overline{)7} = 8 \overline{)7.0} = 8 \overline{)7.00} = 0.875$$

$$\frac{7}{8} = 0.875$$

Find 30% of 70. **21**

$$30\% = \frac{3}{10}$$

So we can find 30% by dividing by 10, then multiplying by 3. $70 \div 10 = 7$
 $7 \times 3 = 21$

Increase 60 by 20%

$$100\% = 60$$

$$20\% = 12$$

$$60 + 12 = 72$$

Decrease 80 by 45%

$$100\% = 80$$

$$45\% = 36$$

$$80 - 36 = 44$$

CORE	GOOD TO KNOW...	HOW TO....
<p>Operations with Fractions</p> <ul style="list-style-type: none"> - Add/ subtract fractions by finding equivalent fractions with the same denominator - Multiply fractions by multiplying the numerators together and the denominators together - Divide fractions by following the KFC rule: keep the first fraction as it is, flip the second fraction around so the numerator becomes the denominator and change the sign from a divide to times. 	<p>Direct proportion means that one quantity increases at the same rate as the other.</p> <ul style="list-style-type: none"> - If one banana costs 20p, three bananas will cost 60p etc. The amount of bananas increase by x3 and the cost also increases by x3 so both are in direct proportion. 	$2\frac{3}{4} = \frac{(4 \times 2) + 3}{4} = \frac{8 + 3}{4} = \frac{11}{4}$ <p>Mixed Number Improper Fraction</p> <p>Write $\frac{7}{8}$ as a decimal.</p> $\frac{7}{8} = 8 \overline{)7.000} = 8 \overline{)7.000} = 0.875$ $\frac{7}{8} = 0.875$
<p>Ratios</p> <p>A unit ratio is a ratio written in the form 1 : n, where n is a number</p> <p>Keywords</p> <p>Fraction - A fraction represents a part of a whole.</p> <p>Decimal - A number with a decimal point in it.</p> <p>Percentage - A relative value indicating hundredth parts of any quantity e.g 1% of £100 = £1</p> <p>Numerator - The part of a fraction that is above the line and signifies the number to be divided by the denominator.</p> <p>Denominator - The part of a fraction that is below the line and that functions as the divisor of the numerator.</p> <p>Ratio - A ratio shows how much of one thing there is compared to another.</p> <p>Simple interest - is the interest calculated only on the original amount invested. It is the same each year.</p>	<p>There are 20 students in a class. 6 are male. What percentage of the class is male?</p> <p>Method A: $\frac{6}{20} \times 100\% = 6 \times \frac{100}{20} = 30\%$</p> <p>Method B: $\frac{6}{20} = \frac{30}{100} = 30\%$ Convert to a fraction with denominator 100.</p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid gray; border-radius: 10px; padding: 5px; width: 45%;"> <p>Work out</p> $\frac{3}{4} \times \frac{2}{7} = \frac{3 \times 2}{4 \times 7} = \frac{6}{28} = \frac{3}{14}$ </div> <div style="border: 1px solid gray; border-radius: 10px; padding: 5px; width: 45%;"> <p>Work out</p> $\frac{3}{4} \div \frac{2}{7} = \frac{3}{4} \times \frac{7}{2} = \frac{21}{8} = 2\frac{5}{8}$ </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="border: 1px solid gray; border-radius: 10px; padding: 5px; width: 45%;"> <p>Work out</p> $\frac{3}{4} + \frac{2}{7} = \frac{21}{28} + \frac{8}{28} = \frac{29}{28} = 1\frac{1}{28}$ </div> <div style="border: 1px solid gray; border-radius: 10px; padding: 5px; width: 45%;"> <p>Work out</p> $\frac{3}{4} - \frac{2}{7} = \frac{21}{28} - \frac{8}{28} = \frac{13}{28}$ </div> </div>	<p style="text-align: center; color: green;">Share \$48 in the ratio 3:1:2</p> <ol style="list-style-type: none"> Find the total number of parts $3 + 1 + 2 = 6$ <ol style="list-style-type: none"> Divide the amount by the total number of parts $\$48 \div 6 = \$8 = 1 \text{ part}$ <ol style="list-style-type: none"> Multiply each number in the ratio by the value of 1 part <div style="text-align: center; margin-top: 20px;"> $3 : 1 : 2$ $\\$24 : \\$8 : \\$16$ </div>

CORE

- A letter represents an unknown variable
- Manipulate an equation using inverse operations, e.g. make x the subject of the equation i.e. rearrange the equation so that x is on its own
 - $x + y = 7$ becomes $x = 7 - y$ by subtracting y from both sides
- The letter n is generally used for sequences
- Continue a pictorial or numerical sequence - e.g. the first 4 terms in a sequence are 4, 7, 10, 13 the next term is 16 as the pattern is going up by 3 each time
- Inequalities can be written as an equation or represented on a number line
 - $<$ means less than
 - $>$ means greater than
 - \leq means less than or equal to
 - \geq means greater than or equal to
- Expand - multiply term outside the bracket by all terms inside the brackets to eliminate brackets
- Substitute - replace the given letter with the given value
- Solve - find the exact value of the unknown variable
- Term - is a number in a sequence e.g. 1st term etc
- Inverse - opposite, e.g. inverse of add is subtract
- Expression - Numbers, symbols and operators grouped together e.g. $2x + 3$ is an expression
- Equation - an expression that contains an equals sign
- Identity - an equation that is true no matter what values are chosen
- Formula - a mathematical rule
- Sequence - a list of numbers or objects in a particular order
- Integer - a whole number

GOOD TO KNOW...

- Make x the subject means rearrange the equation so that x is on its own on one side
- Use changing the subject and inverse operations to solve equations
- When multiplying or dividing both sides of an inequality by a negative number the inequality sign reverses
- When we solve equations, we use inverse operations to work out the value of x .

E.g. solve $3x + 4 = 40$

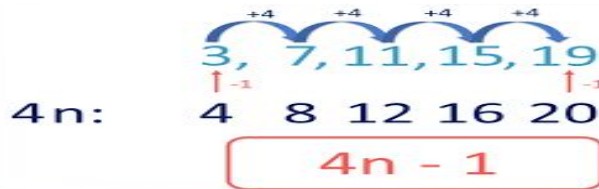
$$\begin{array}{r} -4 \quad -4 \\ 3x = 36 \\ +3 \quad +3 \\ x = 12 \end{array}$$

- The n th term of a sequence is the general rule to work out any term in that sequence.
- Integers solutions can be given for inequalities
E.g. write the integer solutions which satisfy the inequality $1 < x \leq 5$

The integer solutions would be 2, 3, 4 and 5

- To continue a sequence, we need to find the term-to-term rule

E.g. A sequence starts 3, 8, 13, 18, ... Find the next two terms in the sequence. The rule is $+5$ so $18 + 5 = 23$ and $23 + 5 = 28$.



HOW TO....

$$3(a+4) = 3a + 12 \quad 2x(x+y)$$

$$4(a-5) = 4a - 20 = 2x^2 + 2xy$$

Make c the subject

$$A = 3b + 3c$$

$$A - 3b = 3c$$

$$\frac{A - 3b}{3} = \frac{3c}{3}$$

$$\frac{A - 3b}{3} = \frac{3c}{3}$$

$$\frac{A - 3b}{3} = c$$

Subtract $3b$
Divide by 3
Cancel out on the RHS

Problem: $2x - 5 < 1$

Solution: $2x - 5 + 5 < 1 + 5$

$$2x < 6$$

$$\frac{2x}{2} < \frac{6}{2}$$

$$x < 3$$

Inequalities on a Number Line

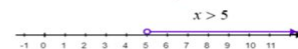
Symbol

Words

Example

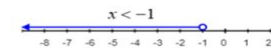
$>$

Greater than



$<$

Less than



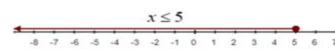
\geq

Greater than or equal to



\leq

Less than or equal to



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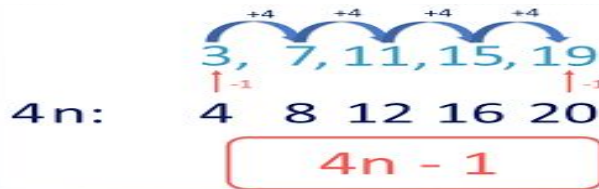
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Divide by 3
Cancel out on the RHS

Problem: $2x - 5 < 1$

Solution: $2x - 5 + 5 < 1 + 5$

$$\begin{array}{r} 2x < 6 \\ \frac{2x}{2} < \frac{6}{2} \\ x < 3 \end{array}$$

Inequalities on a Number Line

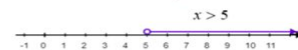
Symbol

Words

Example

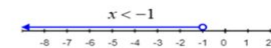
$>$

Greater than



$<$

Less than



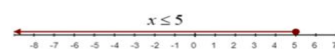
\geq

Greater than or equal to



\leq

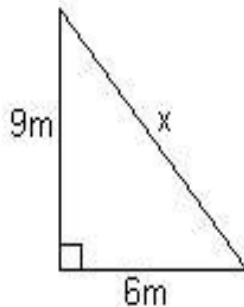
Less than or equal to



CORE

- Angles are measured in degrees
 - Angles on a straight line add up to 180
 - A right angle is 90 degrees
 - Angles around a point add up to 360
 - Angles in a triangle add up to 180
 - Angles in a quadrilateral add up to 360
 - Exterior angles add up to 360
 - Vertically opposite angles are equal
 - Co-interior angles add up to 180
 - Alternate angles are equal
 - Corresponding angles are equal
- Pythagoras theorem is $a^2 + b^2 = c^2$, where c is the longest side
 - Angle notation $\angle ABC$ means the angle at B
 - Parallel - Two straight lines equidistant apart which never meet
 - Polygon - a 2D shape made up of straight lines
 - Equilateral triangle - a triangle with 3 equal angles and sides
 - Isosceles triangle - a triangle with 2 equal angles and sides
 - Right angled triangle - a triangle with one right angle
 - Interior angle - an angle inside a polygon
 - Exterior - an angle outside a polygon
 - Perpendicular - at 90 degrees to a given line
 - Congruent - a shape that is exactly the same shape and size
 - Similar - a shape with the same size angles with all corresponding sides in proportion
 - Hypotenuse - the longest side of a right angled triangle found opposite the right angle
 - Bisect - divide in half exactly
 - Surd - a number that cannot be simplified to remove a square root

GOOD TO KNOW...



$$\begin{aligned} x^2 &= 6^2 + 9^2 \\ x^2 &= 36 + 81 \\ x^2 &= 117 \\ x &= \sqrt{117} \\ x &= 10.8\text{m (1dp)} \end{aligned}$$

	0°	30°	45°	60°	90°
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan(\theta)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

Exterior Angles

The sum of the exterior angles of any polygon is 360° .

The exterior angle of a regular n-sided polygon is $\frac{360^\circ}{n}$

Formula for the sum of Polygon interior angles

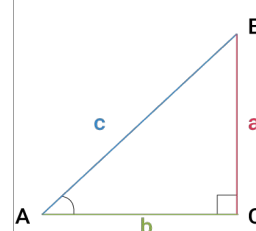
square

4

$$\begin{aligned} \text{sum} &= (n - 2) \times 180^\circ \\ &= (4 - 2) \times 180^\circ \\ &= 360^\circ \end{aligned}$$

HOW TO....

SOH - CAH - TOA



$$\text{sine of } \angle A = \sin A = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{a}{c}$$

$$\text{cosine of } \angle A = \cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{b}{c}$$

$$\text{tangent of } \angle A = \tan A = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{a}{b}$$

Calculate p. Give your answer correct to 2 decimal places.

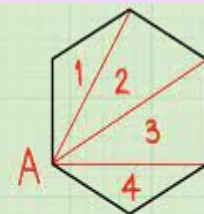
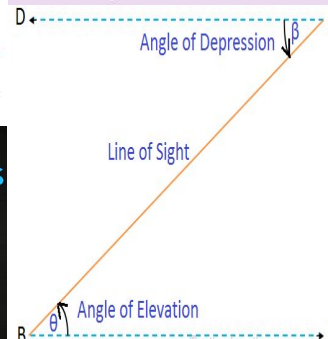
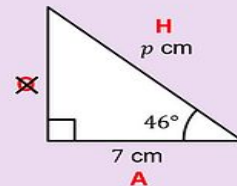
$$\cos \theta = \frac{A}{H}$$

$$\cos 46^\circ = \frac{7}{p}$$

$$p \times \cos 46^\circ = 7$$

$$p = \frac{7}{\cos 46^\circ}$$

$$p = 10.08 \text{ cm}$$



$$4 \times 180^\circ = 720^\circ$$

CORE

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- Perpendicular - at 90 degrees to a given line

- Congruent - a shape that is exactly the same shape and size
- Similar - a shape with the same size angles with all corresponding sides in proportion

GOOD TO KNOW...

- Names of 2D shapes
 - 5 sided - Pentagon
 - 6 sided - Hexagon
 - 7 sided - Heptagon
 - 8 sided - Octagon
 - 9 sided - Nonagon
 - 10 sided - Decagon

- $A <$ is used to label an angle.
- $\angle ABC$ refers to the middle letters so we would be looking at the angle at B shown below.

The figure below also illustrates the angle $\angle ABC$

The outside Angle of a Triangle always equals the sum of the two far away inside angles.

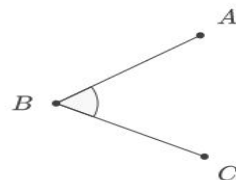
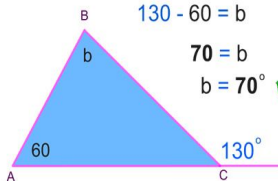
$$\text{Exterior Angle } C^\circ = a + b$$

$$130 = 60 + b$$

$$130 - 60 = b$$

$$70 = b$$

$$b = 70^\circ \checkmark$$



Formula for the sum of Polygon interior angles

square

4

$$\begin{aligned} \text{sum} &= (n - 2) \times 180^\circ \\ &= (4 - 2) \times 180^\circ \\ &= 360^\circ \end{aligned}$$

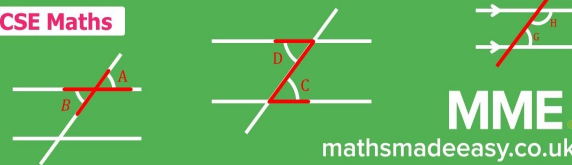
HOW TO....

Geometry

Geometry Basics

Corresponding and Alternate Angles

GCSE Maths



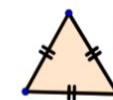
MME

mathsmadeeasy.co.uk

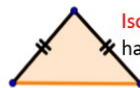
Exterior Angles

The sum of the exterior angles of any polygon is 360° .

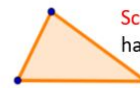
The exterior angle of a regular n-sided polygon is $\frac{360^\circ}{n}$



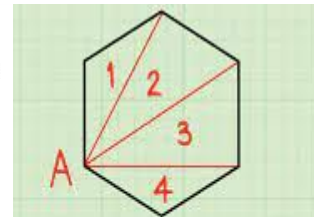
Equilateral Triangle
has three equal sides



Isosceles Triangle
has two equal sides



Scalene Triangle
has no equal sides



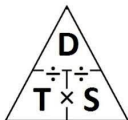
$$4 \times 180^\circ = 720^\circ$$

CORE

GOOD TO KNOW...

HOW TO....

- A linear graph is a straight line
- Quadratic, cubic and reciprocal graphs are curved
- $y = mx + c$ represents a linear graph where m is the gradient and c is the y intercept
- The mid point is halfway between the two given points
- The diameter is double the radius



Keywords

- Linear - when graphed creates a straight line
- Quadratic - one unknown term is squared
- Cubic - one unknown term is cubed
- Equation - an expression that contains an equals sign
- Root - a solution to a quadratic or cubic equation. There can be more than one root
- Origin - the point where the x and y axes intersect
- Axis - the horizontal or vertical number line which intersect to create a coordinate grid
- Gradient - the steepness of a line
- Y -intercept - the point where a line cuts the y axis
- Proportion - a mathematical comparison between two numbers - if the ratios that the two numbers increase/decrease are the same this is direct proportion
- Perpendicular - at 90 degrees to a given line

- Know that a graph axis doesn't have to start at zero but can start at any number using a zigzag between the origin and the first defined number
- Be able to find the equation of a line perpendicular to a given line
- Use the formula to calculate the gradient of a graph

$$m = \frac{\text{difference in } y}{\text{difference in } x}$$

- Substitute values into an equation to formulate a table of values to create a graph
- Understand and interpret distance time graphs, velocity graphs and calculate rates of change
- Calculate area under graph
- Know that a quadratic and cubic equation can have more than 1 solution
- Acceleration = $\frac{\text{change in velocity}}{\text{time}}$
- The equation for a circle with centre $(0, 0)$, is given by the equation $x^2 + y^2 = r^2$ where r is the radius

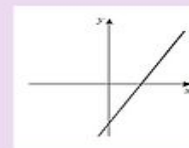
Find the equation of the line that is perpendicular to

$y = -\frac{1}{3}x + 4$ and passes through $(9, -5)$

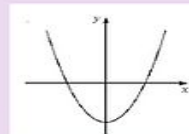
$m = 3$
 $y = 3x + b$
 $-5 = 3(9) + b$
 $-5 = 27 + b$
 $-27 - 27$
 $-32 = b$

$y = 3x - 32$

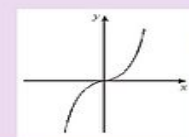
Types of Graphs



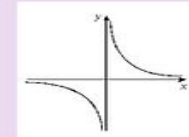
Linear



Quadratic



Cubic



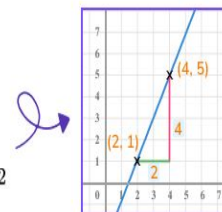
Reciprocal

To calculate the gradient of a straight line through two coordinates (x_1, y_1) and (x_2, y_2) :

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

E.g.

$$m = \frac{5 - 1}{4 - 2} = \frac{4}{2} = 2$$



It can be helpful to think about this formula as: "Change in y divided by change in x " or "Rise over run"

CORE

GOOD TO KNOW...

HOW TO....

- When we order a set of numbers, we need to line them up either:
 - Ascending - smallest value to biggest
 - Descending - biggest to smallest
- Sampling - using a portion of a total population to represent the full population
- Mean - an average calculated by adding all the values and dividing by the total number of values
- Mode - the most common value
- Median - list numbers in numerical order and find the middle value
- Range - the biggest value minus the smallest value
- Outlier - a data point which doesn't fit the trend of the rest of the data

Example: Parking Spaces per House in Hampton Street

Isabella went up and down the street to find out how many parking spaces each house has. Here are her results:

Parking Spaces	Frequency
1	15
2	27
3	8
4	5

What is the mean number of Parking Spaces?

Answer:

$$\begin{aligned} \text{Mean} &= \frac{15 \times 1 + 27 \times 2 + 8 \times 3 + 5 \times 4}{15 + 27 + 8 + 5} \\ &= \frac{15 + 54 + 24 + 20}{55} \\ &= 2.05... \end{aligned}$$

The Mean is **2.05** (to 2 decimal places)

Example:

The following is a frequency table of the score obtained in a mathematics quiz. Find the median score.

Score	0	1	2	3	4
Frequency	3	4	7	6	3

Solution:

Number of scores = 3 + 4 + 7 + 6 + 3 = 23 (odd number)

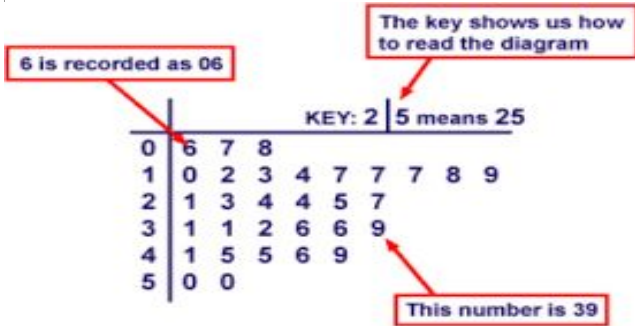
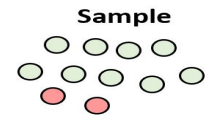
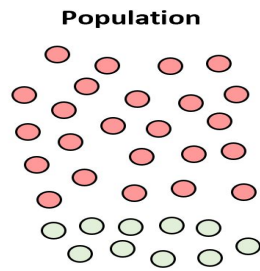
Since the number of scores is odd, the median is at the $\left(\frac{n+1}{2}\right)^{\text{th}} = \left(\frac{23+1}{2}\right)^{\text{th}} = 12^{\text{th}}$ position.

To find out the 12th position, we need to add up the frequencies as shown:

Score	0	1	2	3	4
Frequency	3	4	7	6	3
Position	3	3 + 4 = 7	7 + 7 = 14		

The 12th position is after the 7th position but before the 14th position. So, the median is 2.

Marks scored	Frequency	Mid-point	Frequency × Mid-point
0 - 9	3	$\frac{0+9}{2} = 4.5$	$3 \times 4.5 = 13.5$
10 - 19	5	$\frac{10+19}{2} = 14.5$	$5 \times 14.5 = 72.5$
20 - 29	8	$\frac{20+29}{2} = 24.5$	$8 \times 24.5 = 196$
30 - 39	4	$\frac{30+39}{2} = 34.5$	$4 \times 34.5 = 138$
	n = 20		Total = 420



Goals Scored Over the Last 7 Games

1 3 4 6 6 7 8

mean average **5**

mode most common **6**

median middle **6**

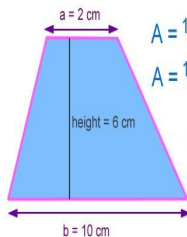
range largest - smallest **7**

CORE

- 2D - a 2 dimension shape (flat, e.g. square, circle)
- 3D - a 3 dimensional shape (solid, e.g. cube, cylinder)
- Area - the space inside a 2D shape measured in squared units
- Area of a rectangle/square = length x width
- Area of a triangle = $\frac{\text{base} \times \text{height (perpendicular)}}{2}$
- Area of a circle = πr^2
- Circumference of a circle = πd
- Area of a trapezium = $\frac{(a+b)h}{2}$
- Volume - the space inside a 3D shape measured in cubed units
- Volume of a cube/cuboid = length x width x height
- Volume of a prism = area of cross section (front face) x length
- Volume of a cylinder = $\pi r^2 \times h$
- Know properties of triangles and quadrilaterals
- Prism - a 3D shape which has the same cross section throughout, e.g. cuboid, cylinder, triangular prism)
- Arc - a curve joining two points on the circumference of a circle
- Sector - a region of a circle bounded by two radii and an arc

GOOD TO KNOW...

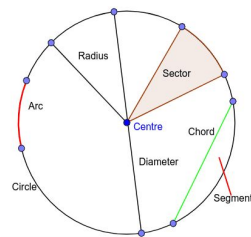
- Plan is the view of a 3D shape when looked at from above
- Elevation is the view of a 3D shape when looked at from the front or the side
- Be able to find the area of a compound shape
- Understand and use bounds
- Surface area is the area of all the faces of a 2D shape added together
- Perpendicular - at an angle of 90 degrees to a given line
- Know that a hemisphere is half a sphere
- Recognise and identify cones, pyramids and frustums
- Identify arcs, sectors and segments



$$A = \frac{1}{2}(a + b) \times h$$

$$A = \frac{1}{2}(2 + 10) \times 6$$

$$A = 36 \text{ cm}^2$$



1m

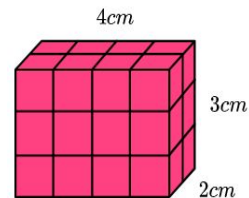


100cm



1 metre squared = 10,000 centimetre squared

HOW TO....



This cuboid is made from 24 unit cubes.

Its volume is

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

$$\text{Volume} = 2 \times 4 \times 3$$

$$\text{Volume} = 24 \text{ cm}^3$$

What is the area of a circle with radius 3cm?



$$\text{Area} = \pi r^2$$

$$= \pi \times 3^2$$

$$= 9\pi \text{ cm}^2$$

$$= 28.3 \text{ cm}^2 \text{ (1.d.p.)}$$

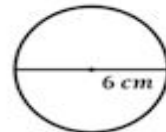
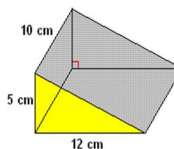
$$V = \frac{(b \times h) \times H}{2}$$

$$V = \frac{(12 \times 5) \times 10}{2}$$

$$V = \frac{60 \times 10}{2}$$

$$V = 30 \times 10$$

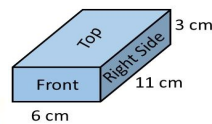
$$V = 300 \text{ cm}^3$$



$$C = \pi d$$

$$= 3.142 \times 6 \text{ cm}$$

$$= 18.85 \text{ cm}$$



Front $6 \times 3 = 18$

Front & Back
(2 × 18) = 36

Right Side $11 \times 3 = 33$

Right & Left Side
(2 × 33) = 66

Top $6 \times 11 = 66$

Top & Bottom
(2 × 66) = 132

Total Surface Area = 234 cm²

CORE

GOOD TO KNOW...

HOW TO....

Perimeter

- Calculated by adding up the length of each of the sides.

Area

- Square/Rectangle = length x width
- Triangle = $\frac{1}{2} \times \text{base} \times \text{height}$
- Measure in squared units, e.g. cm^2 .

Volume

- Cube/cuboid = length x width x height
- Prism = area of cross section (front face) x length
- Measure in cubic units, e.g. cm^3 .

Keywords

Perimeter - The distance around the edge of a shape.

Area - The space inside a 2D shape.

Volume - The volume of a 3D shape is the amount of space inside it.

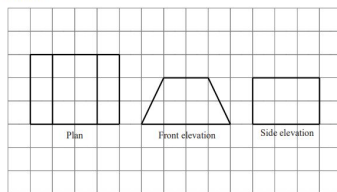
Surface Area - The amount of space covering the outside of a 3D shape.

Perpendicular Height - The line at a right angle to the base line.

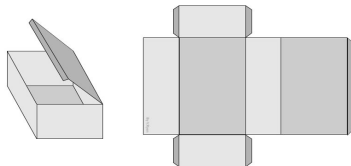
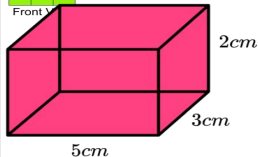
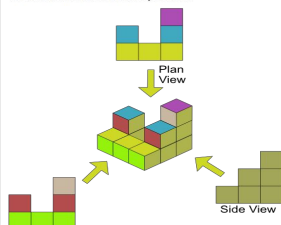
Prism - A 3D shape which has the same cross section throughout, e.g. cuboid, cylinder, triangular prism).

Parallelogram - A quadrilateral with two pairs of parallel sides. Looks like a slanted rectangle.

The diagram shows the plan, front elevation and side elevation of a solid shape, drawn on a centimetre grid.

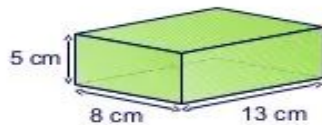


In the space below, draw a sketch of the solid shape. Give the dimensions of the solid on your sketch.

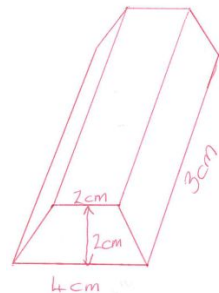


Face	Area
Bottom	$5 \times 3 = 15$
Top	15
Front	$5 \times 2 = 10$
Back	10
Right side	$2 \times 3 = 6$
Left side	6

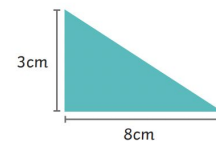
$$\text{Total surface area} = 15 + 15 + 10 + 10 + 6 + 6 = 62\text{cm}^2$$



$$\begin{aligned} \text{Volume of cuboid} &= \text{length} \times \text{width} \times \text{height} \\ &= 5 \times 8 \times 13 \\ &= 520\text{cm}^3 \end{aligned}$$



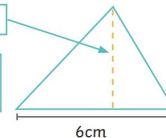
$\text{base} \times \text{perpendicular height} \div 2 = \text{area of a triangle}$



$$\begin{aligned} 8\text{cm} \times 3\text{cm} \div 2 \\ \text{area} = 12\text{cm}^2 \end{aligned}$$

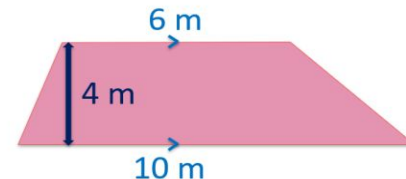
perpendicular height = 5cm

$$\begin{aligned} 6\text{cm} \times 5\text{cm} \div 2 \\ \text{area} = 15\text{cm}^2 \end{aligned}$$



To find the area of a trapezium, add the parallel sides, divide by 2

then multiply by the distance between the parallel sides



$$\begin{aligned} \text{Area} &= \left(\frac{a+b}{2}\right) h = \left(\frac{6+10}{2}\right) \times 4 = 8 \times 4 \\ &= 32\text{m}^2 \end{aligned}$$

CORE

- Transformations - Transformations change the size or position of shapes. There are four types of transformations: reflections, enlargements, rotations, translations.
- Reflection - A shape can be reflected across a line of reflection to create an image, like looking in a mirror. The line of reflection is also called the mirror line. Every point in the image is the same distance from the mirror line as the original shape.
- Rotation - Rotation turns a shape around a fixed point called the centre of rotation. There are three things needed to rotate a shape: the centre of rotation (a coordinate), the angle of rotation (90° , 180° etc.) and the direction of rotation (clockwise or anti-clockwise)
- Translation - A translation moves a shape up, down or from side to side but it does not change its appearance in any other way.
- Enlargement - Enlarging a shape changes its size. The shape can get either bigger or smaller. Two things are needed to enlarge a shape: scale factor ($\times 2$ would make a side twice as big) and the centre of enlargement (a coordinate)
- 3D shapes can be drawn from different viewpoints.
- The plan looks at a shape from above (the birdseye view)
- The front elevation looks at a shape from the front
- The side elevation looks at a shape from the side
- We draw the plan and elevations as 2D shapes.

GOOD TO KNOW...

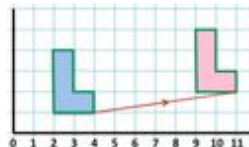
TRANSFORMATIONS

A CHANGE IN THE POSITION OR SIZE OF AN OBJECT

TRANSLATION

Described by a vector

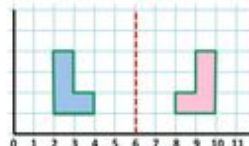
VECTOR $\begin{pmatrix} 7 \\ 1 \end{pmatrix}$



REFLECTION

Described by a mirror line

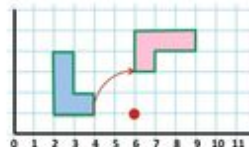
MIRROR LINE $x = 6$



ROTATION

Described by an angle and a centre

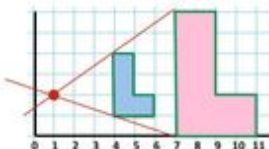
90° CLOCKWISE, CENTRE (6, 1)



ENLARGEMENT

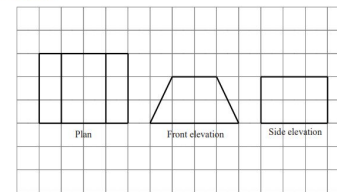
Described by a scale factor and a centre

SCALE FACTOR = 2, CENTRE (1, 2)

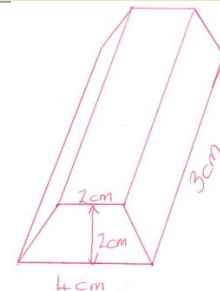


HOW TO....

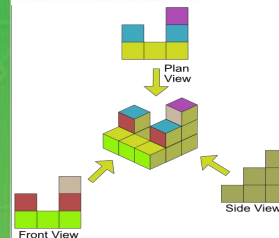
The diagram shows the plan, front elevation and side elevation of a solid shape, drawn on a centimetre grid.



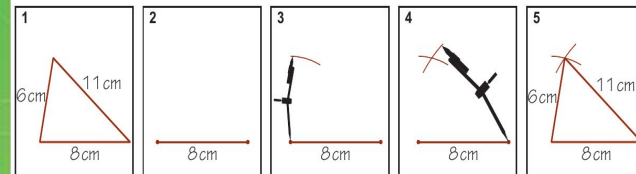
In the space below, draw a sketch of the solid shape. Give the dimensions of the solid on your sketch.



In the space below, draw a sketch of the solid shape. Give the dimensions of the solid on your sketch.



Construct a triangle with sides 11 cm, 8 cm and 6 cm.



- Sketch the triangle first.
- Draw the 8 cm line.
- Open your compasses to 6 cm. Place the point at one end of the 8 cm line. Draw an arc.
- Open your compasses to 11 cm. Draw another arc from the other end of the 8 cm line. Make sure your arcs are long enough to intersect.
- Join the intersection of the arcs to each end of the 8 cm line. Don't rub out your construction marks.