

YEAR 7 — ALGEBRAIC THINKING

Sequences



What do I need to be able to do?

By the end of this unit you should be able to:

- Describe and continue both linear and non-linear sequences
- Explain term rules for linear sequence
- Find missing terms in a linear sequence

Keywords

Sequence: items or numbers put in a pre-decided order

Term: a single number or variable

Position: the place something is located

Rule: instructions that relate two variables

Linear: the difference between terms increases or decreases by the same value each time

Non-linear: the difference between terms increases or decreases in different amounts

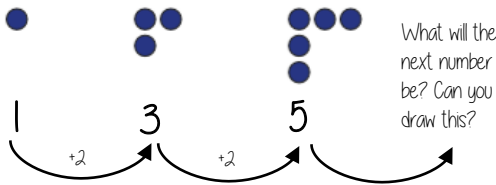
Difference: the gap between two terms

Arithmetic: a sequence where the difference between the terms is constant

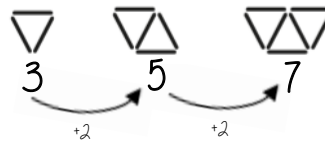
Geometric: a sequence where each term is found by multiplying the previous one by a fixed non zero number

Describe and continue a sequence diagrammatically

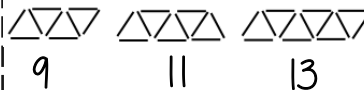
Count the number of circles or lines in each image



Predict and check terms



CHECK — draw the next terms



Predictions:

Look at your pattern and consider how it will increase.

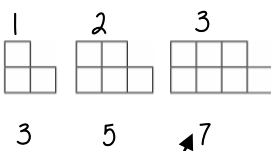
e.g How many lines in pattern 6?

Prediction - 13

If it is increasing by 2 each time - in 3 more patterns there will be 6 more lines

Sequence in a table and graphically

Position: the place in the sequence

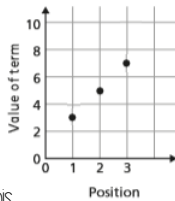


Term: the number or variable (the number of squares in each image)

Position	1	2	3
Term	3	5	7

Because the terms increase by the same addition each time this is **linear** — as seen in the graph

Graphically



"The term in position 3 has 7 squares"

Linear and Non Linear Sequences

Linear Sequences — increase by addition or subtraction and the same amount each time

Non-linear Sequences — do not increase by a constant amount — quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division

Fibonacci Sequence — look out for this type of sequence

0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms

Continue Linear Sequences

7, 11, 15, 19...

How do I know this is a linear sequence?

It increases by adding 4 to each term

How many terms do I need to make this conclusion?

At least 4 terms — two terms only shows one difference not if this difference is constant (a common difference)

How do I continue the sequence?

You continue to repeat the same difference through the next positions in the sequence.

Continue non-linear Sequences

1, 2, 4, 8, 16 ...

How do I know this is a non-linear sequence?

It increases by multiplying the previous term by 2 — this is a geometric sequence because the constant is multiply by 2

How many terms do I need to make this conclusion?

At least 4 terms — two terms only shows one difference not if this difference is constant (a common difference)

How do I continue the sequence?

You continue to repeat the same difference through the next positions in the sequence.

Explain term-to-term rule

How you get from term to term

Try to explain this in full sentences not just with mathematical notation

Use key maths language — doubles, halves, multiply by two, add four to the previous term etc

To explain a whole sequence you need to include a term to begin at ...

The next term is found by tripling the previous term
The sequence begins at 4

4, 12, 36, 108...

First term

YEAR 7 — FRACTIONAL THINKING

Addition and subtraction of fractions

What do I need to be able to do?

By the end of this unit you should be able to:

- Convert between mixed numbers and fractions
- Add/Subtract unit fractions (same denominator)
- Add/Subtract fractions (same denominator)
- Add/Subtract fractions from integers
- Use equivalent fractions
- Add/Subtract any fractions
- Add/Subtract improper fractions and mixed numbers
- Use fractions in algebraic contexts

Keywords

Numerator: the number above the line on a fraction. The top number. Represents how many parts are taken

Denominator: the number below the line on a fraction. The number represent the total number of parts

Equivalent: of equal value

Mixed numbers: a number with an integer and a proper fraction

Improper fractions: a fraction with a bigger numerator than denominator

Substitute: replace a variable with a numerical value

Place value: the value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right

Representing Fractions

$\frac{1}{4}$ is represented in all the images

$1 \div 4$

Mixed numbers and fractions

$\frac{7}{5}$ Improper fraction

$1\frac{2}{5}$ Mixed number

In this model 5 parts make up a whole

Fractions can be bigger than a whole

Odd/Subtract unit fractions Same denominator

$\frac{1}{12} + \frac{1}{12} - \frac{1}{12} = \frac{2}{12}$

$\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$

With the same denominator ONLY the numerator is added or subtracted

Add/Subtract fractions Same denominator

$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$

Sequences

$\frac{1}{3}, 1, 1\frac{2}{3}, 2\frac{1}{3}, 3, \dots$

Represent this on a number line to help

Odd/Subtract from integers

$1 - \frac{2}{6} = \frac{4}{6}$

$3 + \frac{1}{6} = 3\frac{1}{6}$

The denominator indicates the number of parts a whole is made up of

Equivalent fractions Numerator and denominator have the same multiplier

$\frac{2}{3} = \frac{4}{6}$

$\frac{1}{3} = \frac{2}{6}$

Odd/Subtraction fractions (common multiples)

$\frac{3}{5} + \frac{7}{10} = \frac{6}{10} + \frac{7}{10} = \frac{13}{10}$

Addition/Subtraction needs a common denominator

Odd/Subtraction any fractions

$\frac{4}{5} - \frac{2}{3} = \frac{12}{15} - \frac{10}{15} = \frac{2}{15}$

Use equivalent fractions to find a common multiple for both denominators

Odd/Subtraction fractions (improper and mixed)

$2\frac{1}{5} - 1\frac{3}{10} = 2\frac{2}{10} - 1\frac{3}{10} = \frac{22}{10} - \frac{13}{10} = \frac{9}{10}$

- Convert to an improper fraction
- Calculate with common denominator

Partitioning method

$2\frac{1}{5} - 1\frac{3}{10} = 2\frac{2}{10} - 1\frac{3}{10} = 2\frac{2}{10} - 1 - \frac{3}{10} = 1\frac{2}{10} - \frac{3}{10} = \frac{9}{10}$

Fractions in algebraic contexts $p = 5 \quad m = 2$

$k - \frac{5}{8} = 2$

Apply inverse operations: $k = 2 + \frac{5}{8}$

Form expressions with fractions: $b + \frac{7}{9} \rightarrow b + \frac{7}{9}$

Substitution: $\frac{p}{8} + \frac{1}{m} = \frac{5}{8} + \frac{1}{2}$

Fractions and decimals

Example: $\frac{6}{10} + 0.3 = 0.6 + 0.3$

$\frac{1}{10} = 0.1$

$\frac{1}{100} = 0.01$

Remember to use equivalent fractions and common denominators

YEAR 7 — LINES AND ANGLES

Constructing, measuring and using geometric notation

What do I need to be able to do?

By the end of this unit you should be able to:

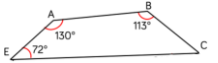
- Use letter and labelling conventions
- Draw and measure line segments and angles
- Identify parallel and perpendicular lines
- Recognise types of triangle
- Recognise types of quadrilateral
- Identify polygons
- Construct triangles (SAS, SSS, ASA)
- Draw Pie charts

Keywords

- Polygon:** A 2D shape made with straight lines
- Scalene triangle:** a triangle with all different sides and angles
- Isosceles triangle:** a triangle with two angles the same size and two sides the same size
- Right-angled triangle:** a triangle with a right angle
- Frequency:** the number of times a data value occurs
- Sector:** part of a circle made by two radii touching the centre
- Rotation:** turn in a given direction
- Protractor:** equipment used to measure angles
- Compass:** equipment used to draw arcs and circles

Letter and labelling convention

The letter in the middle is the angle
The arc represents the angle

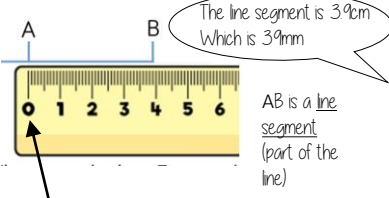


Angle Notation: three letters ABC
This is the angle at B = 113°

Line Notation: two letters EC
The line that joins E to C

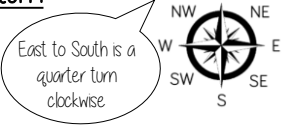
Draw and measure line segments

Conversions 1cm = 10mm, 1m = 100cm



Make sure the start of the line is at 0.

Angles as measures of turn



Clockwise

Anti-Clockwise



Quarter Turn
90°
Clockwise

Half Turn
180°

Three-quarter Turn
270°
Anti-Clockwise

Full Turn
360°

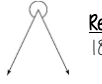
Classify angles



Acute Angles
0° < angle < 90°



Obtuse
90° < angle < 180°

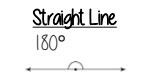


Reflex
180° < angle < 360°



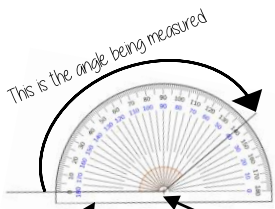
Right Angles
90°

Right angle notation



Straight Line
180°

Measure angles to 180°



The base line follows the line segment

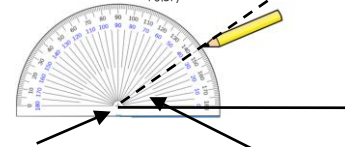
Make sure the cross is at the point the two lines meet

Read from 0° on the base line. Remember to use estimation. This is an obtuse angle so between 90° and 180°

Draw angles up to 180°

Draw a 35° angle

Make a mark at 35° with a pencil. And join to the angle point (use a ruler)

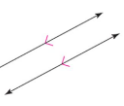


Make sure the cross is at the end of the line (where you want the angle)

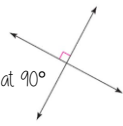
The angle

Parallel and Perpendicular lines

Parallel lines
Straight lines that never meet (Have the same gradient)



Perpendicular lines
Straight lines that meet at 90°



Angles over 180°

Use your knowledge of straight lines 180° and angles around a point 360°

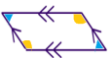
360° - smaller angle = reflex angle



Measure the smaller angle first (less than 180°)

Properties of Quadrilaterals

Square
All sides equal size
All angles 90°
Opposite sides are parallel



Parallelogram
Opposite sides are parallel
Opposite angles are equal
Co-interior angles

Rectangle
All angles 90°
Opposite sides are parallel



Trapezium
One pair of parallel lines

Rhombus
All sides equal size
Opposite angles are equal

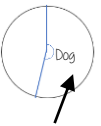


Kite
No parallel lines
Equal lengths on top sides
Equal lengths on bottom sides
One pair of equal angles

Draw Pie Charts

Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

$\frac{32}{60}$ "32 out of 60 people had a dog"



This fraction of the 360 degrees represents dogs

Use a protractor to draw. This is 192°

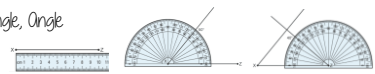
$\frac{32}{60} \times 360 = 192°$

Polygons

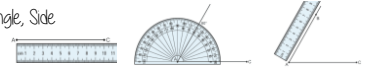
- 3 - Triangle
- 4 - Quadrilateral
- 5 - Pentagon
- 6 - Hexagon
- 7 - Heptagon
- 8 - Octagon
- 9 - Nonagon
- 10 - Decagon

SAS, SSS, ASA constructions

Side, Angle, Angle



Side, Angle, Side



Side, Side, Side



If all the sides and angles are the same, it is a **regular** polygon

YEAR 7 — LINES AND ANGLES

Geometric reasoning

What do I need to be able to do?

By the end of this unit you should be able to:

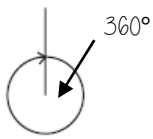
- Understand/use the sum of angles at a point
- Understand/use the sum of angles on a straight line
- Understand/use equality of vertically opposite angles
- Know and apply the sum of angles in a triangle
- Know and apply the sum of angles in a quadrilateral

Keywords

- Vertically Opposite:** angles formed when two or more straight lines cross at a point
- Interior Angles:** angles inside the shape
- Sum:** total, add all the interior angles together
- Convex Quadrilateral:** a four-sided polygon where every interior angle is less than 180°
- Concave Quadrilateral:** a four-sided polygon where one interior angle exceeds 180°
- Polygon:** a 2D shape made with straight lines
- Scalene triangle:** a triangle with all different sides and angles
- Isoceles triangle:** a triangle with two angles the same size and two angles the same size
- Right-angled triangle:** a triangle with a right angle

Sum of angles at a point

The sum of angles around a point is 360°



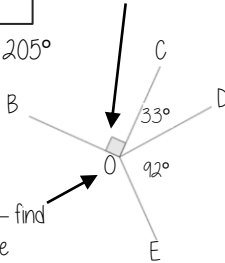
Find angle BOE

$$90^\circ + 33^\circ + 92^\circ = 205^\circ$$

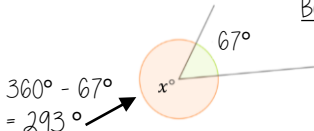
$$360^\circ - 205^\circ$$

$$BOE = 155^\circ$$

Angle notation — 90°

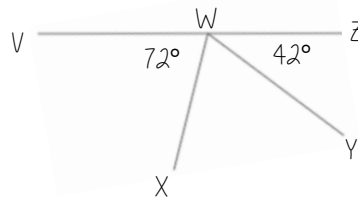


Angle notation — find this missing angle



Sum of angles on a straight line

Adjacent angles that share a common point on a line add up to 180°

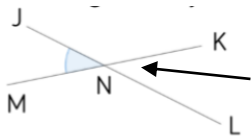


Find angle XWY

$$72^\circ + 42^\circ = 114^\circ$$

$$180^\circ - 114^\circ = 66^\circ$$

Vertically opposite angles

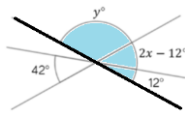


Angle JNM is vertically opposite to angle KNL

$$JNM = KNL$$

Vertically opposite angles are the same

Other angle rules still apply
Look for straight line sums and angles around a point

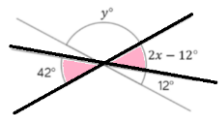


Form equations with information from diagrams

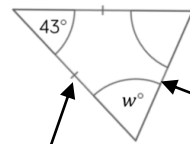
$$2x - 12 = 42$$

$$2x = 54$$

$$x = 27^\circ$$



Sum of angles in triangles



The two base angles will be the same size

Look at triangle notation
This indicates an isosceles triangle

$$\therefore 180 - 43 = 137$$

$$137 \div 2 = 68.5^\circ$$

A triangle can only have ONE right angle

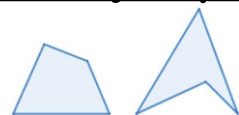
Sum of interior angles in a triangle = 180°



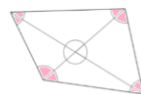
Have a go!
Tearing the corners from triangles forms a straight line which is therefore 180°

Sum of angles in quadrilaterals

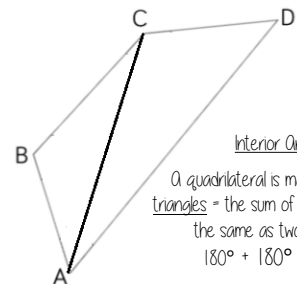
Sum of interior angles in a quadrilateral = 360°



Convex Quadrilateral
Concave Quadrilateral



Interior angles are those that make up the perimeter (outline) of the shape

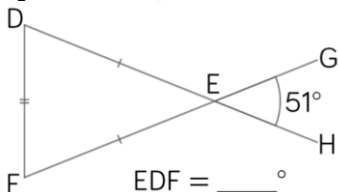


Interior Angles

A quadrilateral is made up of two triangles = the sum of interior angles is the same as two triangles
 $180^\circ + 180^\circ = 360^\circ$

Angle Problems

Split up the problem into chunks and explain your reasoning at each point using angle notation



1. Angle DEF = 51° because it is a vertically opposite angle DEF = GEH
2. Triangle DEF is isosceles (triangle notation) \therefore EDF = EFD and the sum of interior angles is 180°
 $180^\circ - 51^\circ = 129^\circ$ $129^\circ \div 2 = 64.5^\circ$
3. Angle EDF = 64.5°

Keep working out clear and notes together

YEAR 7 — REASONING WITH NUMBER

Developing number sense

What do I need to be able to do?

By the end of this unit you should be able to:

- Know and use mental addition/ subtraction
- Know and use mental multiplication/ division
- Know and use mental arithmetic for decimals
- Know and use mental arithmetic for fractions
- Use factors to simplify calculations
- Use estimation to check mental calculations
- Use number facts
- Use algebraic facts

Keywords

Commutative: changing the order of the operations does not change the result

Associative: when you add or multiply you can do so regardless of how the numbers are grouped

Dividend: the number being divided

Divisor: the number we divide by


Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

Equation: a mathematical statement that two things are equal

Quotient: the result of a division

Mental methods for addition/ subtraction

Addition is commutative Subtraction the order has to stay the same



$6 + 3 = 3 + 6$


The order of addition does not change the result

$360 - 147 = 360 - 100 - 40 - 7$

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction

Mental methods for multiplication/ division

Multiplication is commutative Partitioning can help multiplication



$2 \times 4 = 4 \times 2$

The order of multiplication does not change the result

$24 \times 6 = 20 \times 6 + 4 \times 6$
 $= 120 + 24$
 $= 144$

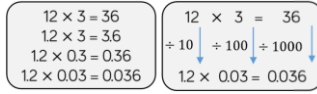
Division is not associative

Chunking the division can help $4000 \div 25$
 "How many 25's in 100" then how many chunks of that in 4000.

Mental methods for decimals

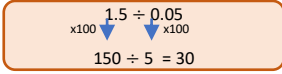
Multiplying by a decimal < 1 will make the original value smaller e.g. $0.1 = \div 10$

Methods for multiplication 12×0.03

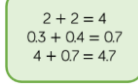


Methods for division $15 \div 0.05$

Multiply by powers of 10 until the divisor becomes an integer

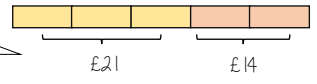


Methods for addition $2.3 + 2.4$

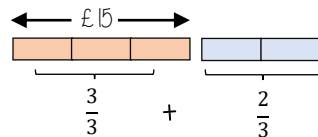


Mental methods for fractions Use bar models where possible

I've spent $\frac{2}{5}$ of my money I have £21 left



How much did they have to begin with?



What is $\frac{5}{3}$ of £15?

Using factors to simplify calculations

30×16

$10 \times 3 \times 4 \times 4$ $10 \times 3 \times 2 \times 8$

$2 \times 5 \times 3 \times 2 \times 2 \times 2 \times 2$ $16 \times 10 \times 3$

Multiplication is commutative
 Factors can be multiplied in any order

Estimation

Estimations are useful — especially when using fractions and decimals to check if your solution is possible.

Most estimations round to 1 significant figure

Estimations are useful — especially when using fractions and decimals to check if your solution is possible.

$210 + 899 < 1200$

This is true because even if both numbers were rounded up, they would reach $300 + 900$.

The correct estimation would be $200 + 900 = 1100$.

Number facts

Use $124 \times 5 = 620$

For multiplication, each value that is multiplied or divided by powers of 10 needs to happen to the result

$620 \div 124 = 50$

For division you must consider the impact of the divisor becoming smaller or bigger.

Smaller — the answer will be bigger (It is being shared into less parts)

Bigger — the answer will be smaller (It is being shared into more parts)

Algebraic facts

$2a + 2b = 10$ Everything $\times 2$

$0.1a + 0.1b = 0.5$ Everything $\div 10$

$a + b = 5$

$a + b + 2 = 7$ Add 2 to the total

The unknown quantity isn't changing but the variables change what is done to give the result

YEAR 7 — REASONING WITH NUMBER

Sets and probability

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify and represent sets
- Interpret and create Venn diagrams
- Understand and use the intersection of sets
- Understand and use the union of sets
- Generate sample spaces for single events
- Calculate the probability of a single event
- Understand and use the probability scale

Keywords

Set: collection of things
Element: each item in a set is called an element
Intersection: the overlapping part of a Venn diagram ($A \cap B$)
Union: two ellipses that join ($A \cup B$)
Mutually Exclusive: events that do not occur at the same time
Probability: likelihood of an event happening
Bias: a built-in error that makes all values wrong (unequal) by a certain amount, e.g. a weighted dice
Fair: there is zero bias, and all outcomes have an equal likelihood
Random: something happens by chance and is unable to be predicted

Identify and represent sets

The **universal set** has this symbol ξ — this means **EVERYTHING** in the Venn diagram is in this set

A set is a collection of things — you write sets inside curly brackets { }

$\xi = \{\text{the numbers between 1 and 50 inclusive}\}$

My sets can include every number between 1 and 50 including those numbers

$A = \{\text{Square numbers}\}$
 $A = \{1, 4, 9, 16, 25, 36, 49\}$

All the numbers in set A are square number and between 1 and 50

Interpret and create Venn diagrams

Mutually exclusive sets
 The two sets have nothing in common
 No overlap

Union of sets
 The two sets have some elements in common — they are placed in the intersection

Subset
 All of set B is also in Set A so the ellipse fits inside the set

The box
 Around the outside of every Venn diagram will be a box. If an element is not part of any set it is placed outside an ellipse but inside the box

Intersection of sets

Elements in the intersection are in set A AND set B

The notation for this is $A \cap B$

$\xi = \{\text{the numbers between 1 and 15 inclusive}\}$
 $A = \{\text{Multiples of 5}\}$ $B = \{\text{Multiples of 3}\}$

The element in $A \cap B$ is 15

In this example there is only one number that is both a multiple of 3 and a multiple of 5 between 1 and 15

Union of sets

Elements in the union could be in set A OR set B

The notation for this is $A \cup B$

This Venn shows the **number of elements** in each set

There are 7 elements that are either a multiple of 5 OR a multiple of 3 between 1 and 15

The elements in $A \cup B$ are 5, 10, 15, 3, 9, 6, 12

Sample space — for single events

A sample space for rolling a six-sided dice is $S = \{1, 2, 3, 4, 5, 6\}$

A sample space for this spinner is $S = \{\text{Pink, Blue, Yellow}\}$

You only need to write each element once in a sample space diagram

- A Sample space represents a possible outcome from an event
- They can be interpreted in a variety of ways because they do not tell you the probability

Probability of a single event

Probability = $\frac{\text{number of times event happens}}{\text{total number of possible outcomes}}$

$P(\text{Blue}) = \frac{4}{10}$ ← There are 4 blue sectors
 ← There are 10 sectors overall

$= \frac{2}{5}$

Probability notation $P(\text{event})$

Probability can be a fraction, decimal or percentage value

$\frac{4}{10} = \frac{40}{100} = 0.40 = 40\%$

Probability is always a value between 0 and 1

The probability scale

Impossible 0 or 0% Even chance 0.5, $\frac{1}{2}$ or 50% Certain 1 or 100%

The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)

There are 2 pink and 2 yellow balls, so they have the same probability

There are 5 possible outcomes So 5 intervals on this scale, each interval value is $\frac{1}{5}$

Sum of probabilities

Probability is always a value between 0 and 1

The probability of getting a blue ball is $\frac{1}{5}$
 \therefore The probability of **NOT** getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

$P(\text{white chocolate}) = 1 - 0.15 - 0.35 = 0.5$

YEAR 7 — REASONING WITH NUMBER

Prime numbers and Proof

What do I need to be able to do?

By the end of this unit you should be able to:

- Find and use multiples
- Identify factors of numbers and expressions
- Recognise and identify prime numbers
- Recognise square and triangular numbers
- Find common factors including HCF
- Find common multiples including LCM

Keywords

Multiples: found by multiplying any number by positive integers
Factor: integers that multiply together to get another number.
Prime: an integer with only 2 factors
Conjecture: a statement that might be true (based on reasoning) but is not proven
Counterexample: a special type of example that disproves a statement
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
HCF: highest common factor (biggest factor two or more numbers share)
LCM: lowest common multiple (the first time the times table of two or more numbers match)

Multiples The "times table" of a given number

All the numbers in this lists below are multiples of 3

3, 6, 9, 12, 15...

$3x, 6x, 9x \dots$

This list continues and doesn't end

Non example of a multiple

45 is not a multiple of 3 because it is 3×15

Not an integer

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Factors

Arrays can help represent factors

Factors of 10: 1, 2, 5, 10

10×1 or 1×10

5×2 or 2×5

The number itself is always a factor

Factors and expressions

Factors of $6x$: $6, x, 1, 6x, 2x, 3, 3x, 2$

$6x \times 1$ OR $6 \times x$

$2x \times 3$

$3x \times 2$

Prime numbers

- Integer
- Only has 2 factors
- and itself

The first prime number

The only even prime number

2

Learn or how-to quick recall...

2, 3, 5, 7, 11, 13, 17, 19, 23, 29...

Square and triangular numbers

Square numbers

Representations are useful to understand a square number n^2

1, 4, 9, 16, 25, 36, 49, 64 ...

odd, even, odd

Triangular numbers

Representations are useful — an extra counter is added to each new row

Add two consecutive triangular numbers and get a square number

1, 3, 6, 10, 15, 21, 28, 36, 45...

Common factors and HCF

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF — Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

Common factors (factors of both numbers): 1, 2, 3, 6

HCF = 6

6 is the biggest factor they share

Common multiples and LCM

Common multiples are multiples two or more numbers share

LCM — Lowest common multiple

LCM of 9 and 12

9: 9, 18, 27, 36, 45, 54

12: 12, 24, 36, 48, 60

LCM = 36

The first time their multiples match

Comparing fractions

Compare fractions using a LCM denominator

$\frac{3}{5}$ and $\frac{7}{10}$

$\frac{6}{10}$ and $\frac{7}{10}$

Product of prime factors

Multiplication part-whole models

30: 2, 15; 3, 10; 5, 6

15: 5, 3; 10: 2, 5; 6: 2, 3

All three prime factor trees represent the same decomposition

Multiplication is commutative

$30 = 2 \times 3 \times 5$

Multiplication of prime factors

Using prime factors for predictions

e.g. 60: 30×2 , $2 \times 3 \times 5 \times 2$

150: 30×5 , $2 \times 3 \times 5 \times 5$

Conjectures and counterexamples

Conjecture

1, 2, 4, ...

The numbers in the sequence are doubling each time.

A pattern that is noticed for many cases

Counterexamples

This sequence isn't doubling it is adding 2 each time

Only one counterexample is needed to disprove a conjecture

YEAR 7 — ALGEBRAIC THINKING...

Algebraic notation

What do I need to be able to do?

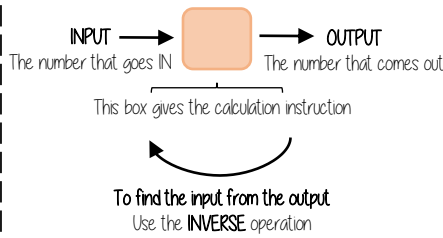
By the end of this unit you should be able to:

- Be able to use inverse operations and "operation families".
- Be able to substitute into single and two step function machines.
- Find functions from expressions.
- Form sequences from expressions.
- Represent functions graphically.

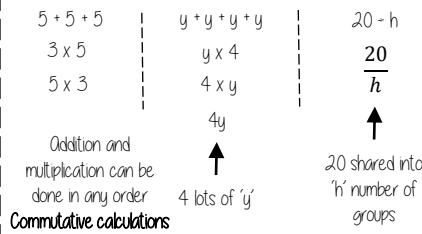
Keywords

- Function:** a relationship that instructs how to get from an input to an output.
- Input:** the number/ symbol put into a function.
- Output:** the number/ expression that comes out of a function.
- Operation:** a mathematical process.
- Inverse:** the operation that undoes what was done by the previous operation. (The opposite operation)
- Commutative:** the order of the operations do not matter.
- Substitute:** replace one variable with a number or new variable.
- Expression:** a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
- Evaluate:** work out.
- Linear:** the difference between terms increases or decreases by the same value each time.
- Sequence:** items or numbers put in a pre-decided order.

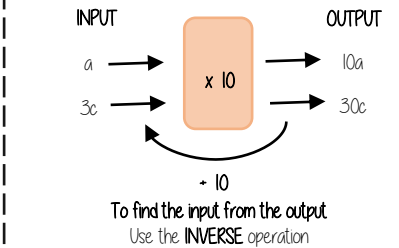
Single function machines



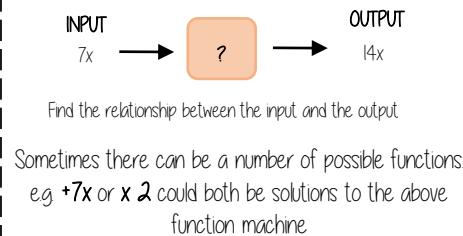
Using letters to represent numbers



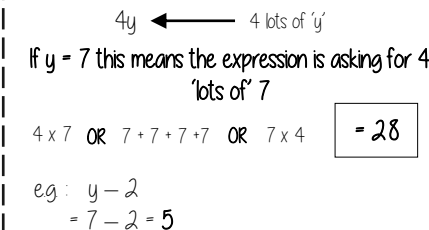
Single function machines (algebra)



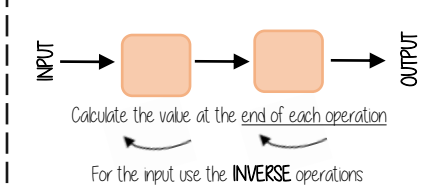
Find functions from expressions



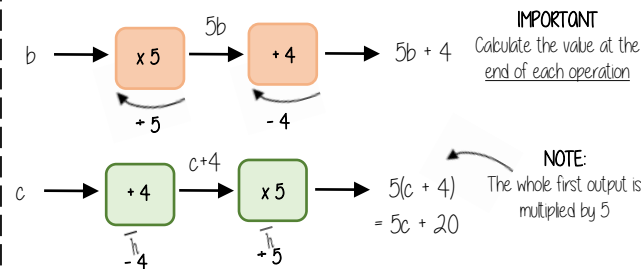
Substitution into expressions



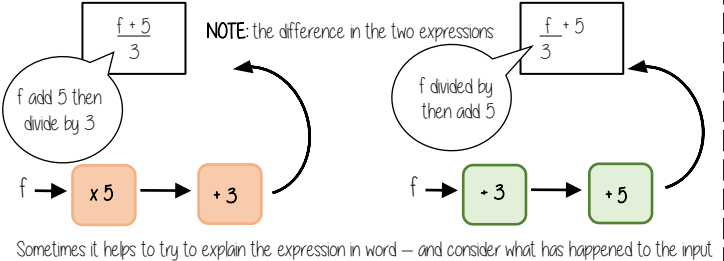
Two step function machines



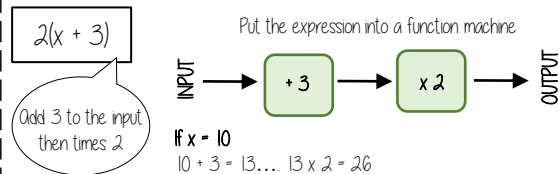
Two step function machines (algebra)



Find functions from expressions



Substitution into an expression



Representing functions graphically

Take the function and generate a sequence $2(x + 3)$



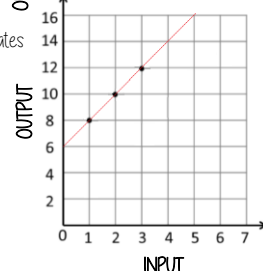
To represent graphically the input becomes x co-ordinates and the output becomes y co-ordinates

$$y = 2(x + 3)$$

INPUT (x)	1	2	3
OUTPUT (y)	8	10	12

This becomes a co-ordinate pair (2, 10) to plot on a graph

Not all graphs will be linear only those with an integer value for x. Powers and fractions generate differently shaped graphs.



NOTE: Because this is a linear graph you can predict other values

Forming a sequence

INPUT	1	2	3
OUTPUT	8	10	12

The substitution is the 'input' value. The OUTPUT becomes the sequence.

YEAR 7 — ALGEBRAIC THINKING

Equality and Equivalence

What do I need to be able to do?

By the end of this unit you should be able to:

- Form and solve linear equations
- Understand like and unlike terms
- Simplify algebraic expressions

Keywords

- Equality:** two expressions that have the same value
- Equation:** a mathematical statement that two things are equal
- Equals:** represented by '=' symbol — means the same
- Solution:** the set or value that satisfies the equation
- Solve:** to find the solution
- Inverse:** the operation that undoes what was done by the previous operation (The opposite operation)
- Term:** a single number or variable
- Like:** variables that are the same are 'like'
- Coefficient:** a multiplicative factor in front of a variable e.g. $5x$ (5 is the coefficient, x is the variable)
- Expression:** a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

Equality

$$2 + 14 = 5 + 5 + 6$$

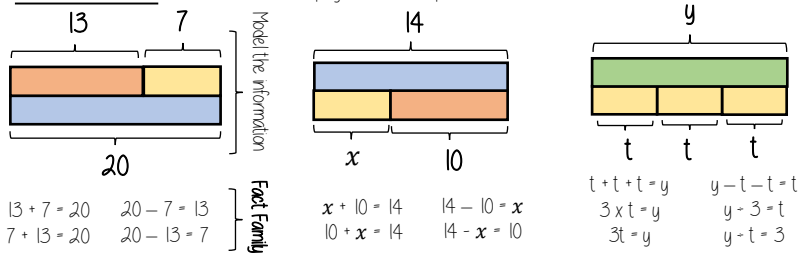


Saying it out loud sometimes helps you to understand equality

The sum on the left has the same result as the sum on the right

Fact Families

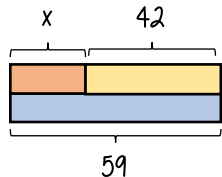
Use a bar model to display the relationships between terms and numbers



Solve one step equations (+/-)

There is more to this than just spotting the answer

$$x + 42 = 59$$



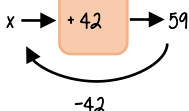
$$x + 42 = 59$$

$$42 + x = 59$$

$$59 - x = 42$$

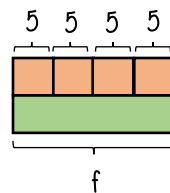
$$59 - 42 = x$$

Don't forget you know how to use function machines



Solve one step equations (x/÷)

$$\frac{f}{4} = 5$$



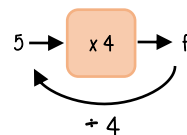
$$f - 4 = 5$$

$$f - 5 = 4$$

$$5 \times 4 = f$$

$$4 \times 5 = f$$

Don't forget you know how to use function machines



Like and unlike terms

Like terms are those whose variables are the same

♥ and 3♥ are like terms
the variable is the same

★ and 3♥ are unlike terms
the variables are NOT the same

Examples and non-examples

Like terms

$y, 7y$
 $2x^2, x^2$
 $ab, 10ba$
 $5, -2$

Un-like terms

$y, 7x$
 $2x^2, 2c^2$
 $ab, 10a$
 $5, -2t$

Note here ab and ba are commutative operations, so are still like terms

Equivalence

Check equivalence by substitution

e.g. $m = 10$

$$5m$$

$$5 \times 10$$

$$= 50$$

$$2 \times 2m$$

$$2 \times (2 \times 10)$$

$$= 2 \times 20$$

$$= 40$$

$$7m - 3m$$

$$(7 \times 10) - (3 \times 10)$$

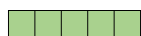
$$= 70 - 30$$

$$= 40$$

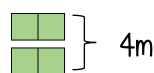
Equivalent expressions

Repeat this with various values for m to check

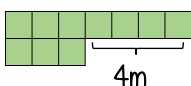
$$5m$$



$$2 \times 2m$$



$$7m - 3m$$



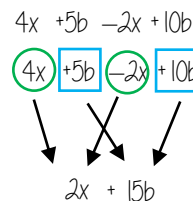
Collecting like terms \equiv symbol

The \equiv symbol means equivalent to

It is used to identify equivalent expressions

Collecting like terms

Only like terms can be combined



Common misconceptions

$$2x + 3x^2 + 4x \equiv 6x + 3x^2$$

Although they both have the x variable x^2 and x terms are unlike terms so cannot be collected

YEAR 7 — PLACE VALUE AND PROPORTION

Ordering integers and decimals

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand place value and the number system including decimals
- Understand and use place value for decimals, integers and measures of any size
- Order number and use a number line for positive and negative integers, fractions and decimals;
- use the symbols $=$, \neq , \leq , \geq
- Work with terminating decimals and their corresponding fractions
- Round numbers to an appropriate accuracy
- Describe, interpret and compare data distributions using the median and range

Keywords

- Approximate:** To estimate a number, amount or total often using rounding of numbers to make them easier to calculate with
- Integer:** a whole number that is positive or negative
- Interval:** between two points or values
- Median:** A measure of central tendency (middle, average) found by putting all the data values in order and finding the middle value of the list
- Negative:** Any number less than zero, written with a minus sign
- Place holder:** We use 0 as a place holder to show that there are none of a particular place in a number
- Place value:** The value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right
- Range:** The difference between the largest and smallest numbers in a set
- Significant figure:** A digit that gives meaning to a number. The most significant digit (figure) in an integer is the number on the left. The most significant digit in a decimal fraction is the first non-zero number after the decimal point

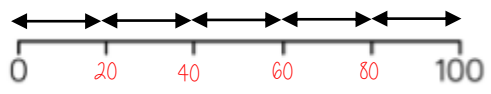
Integer Place Value

Billions			Millions			Thousands			Ones		
H	T	O	H	T	O	H	T	O	H	T	O
		3	1	4	8	0	3	3	0	2	9

Placeholder

Three billion, one hundred and forty eight million, thirty three thousand and twenty nine
 1 billion 1,000,000,000
 1 million 1,000,000

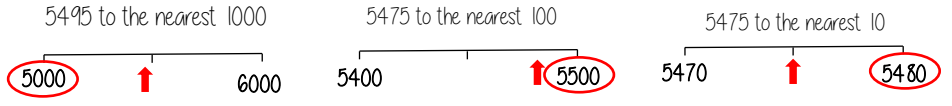
Intervals on a number line



Divide the difference by the number of intervals (gaps).
 Eg $100 \div 5 = 20$

Rounding to the nearest power of ten

If the number is halfway between we "round up"



Compare integers using $<$, $>$, $=$, \neq

- $<$ less than: Two and a half million \leq 2 500 000
- $>$ greater than: 300 000 000 \leq Three billion
- $=$ equal to: Six thousand and eighty \leq 68 000
- \neq not equal to

Range Spread of the values

Difference between the biggest and smallest
 3 9 8 12
 Range: Biggest value - Smallest value
 $12 - 3 = 9$
 Range = 9

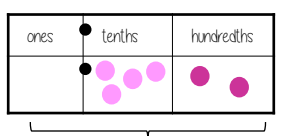
Median The middle value

Example 1: Median: put the in order 3 4 8 9 12
 4 3 9 8 12 find the middle number 3 4 **8** 9 12

Example 2: Median: put the in order 150 154 148 137 148 **150 154** 158 160
 137 160 158 There are 2 middle numbers Find the midpoint 152

Decimals

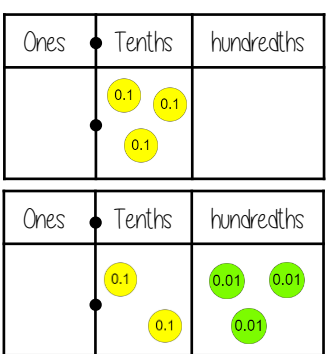
We say "nought point five two"
 Five tenths and two hundredths



0 ones, 5 tenth and 2 hundredths
 $0 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.01 + 0.01$
 $= 0 + 0.5 + 0.02$
 $= 0.52$

Comparing decimals

Which the largest of 0.3 and 0.23?

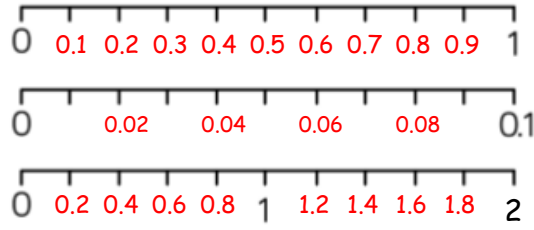


$0.3 > 0.23$
 "There are more counters in the furthest column to the left"

0.30 }
 0.23 }
 Comparing the values both with the same number of decimal places is another way to compare the number of tenths and hundredths

Decimal intervals on a number line

One whole split into 10 parts makes tenths = 0.1
 One tenth split into 10 parts makes hundredths = 0.01



Round to 1 significant figure

- 370 to 1 significant figure is 400
- 37 to 1 significant figure is 40
- 37 to 1 significant figure is 4
- 0.37 to 1 significant figure is 0.4
- 0.00000037 to 1 significant figure is 0.0000004

Round to the first non zero number

YEAR 7 — PLACE VALUE AND PROPORTION... FDP equivalence

What do I need to be able to do?

By the end of this unit you should be able to:

- Convert fluently between fractions, decimals & percentages

Keywords

Fraction: how many parts of a whole we have

Decimal: a number with a decimal point used to separate ones, tenths, hundredths etc.

Percentage: a proportion of a whole represented as a number between 0 and 100

Place value: the numerical value that a digit has decided by its position in the number

Placeholder: a number that occupies a position to give value

Interval: a range between two numbers

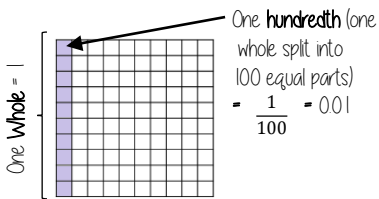
Tenth: one whole split into 10 equal parts

Hundredth: one whole split into 100 equal parts

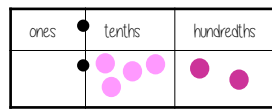
Sector: a part of a circle between two radius (often referred to as looking like a piece of pie)

Recurring: a decimal that repeats in a given pattern

Tenths and hundredths

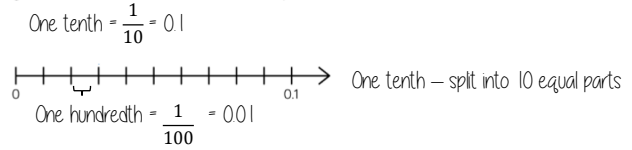
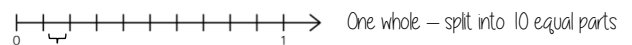


One tenth (one whole split into 10 equal parts) = $\frac{1}{10} = 0.1$

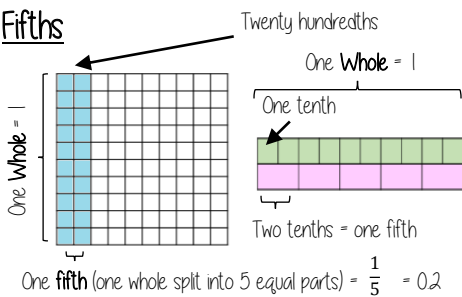


0 ones, 5 tenths and 2 hundredths
 $0 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.01 + 0.01$
 $= 0 + 0.5 + 0.02$
 $= 0.52$

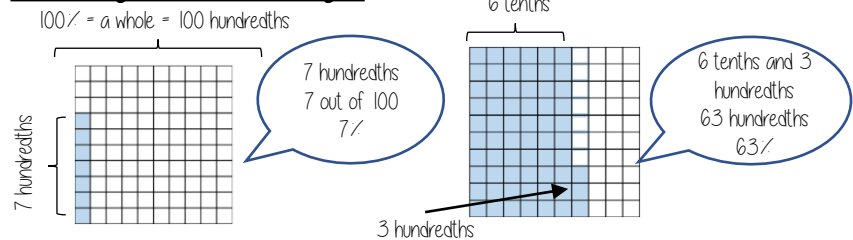
On a number line



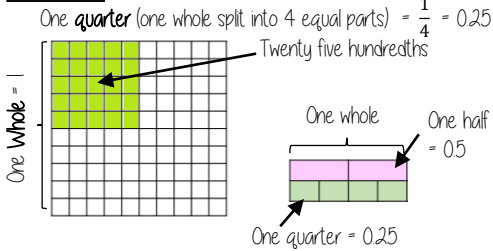
Fifths



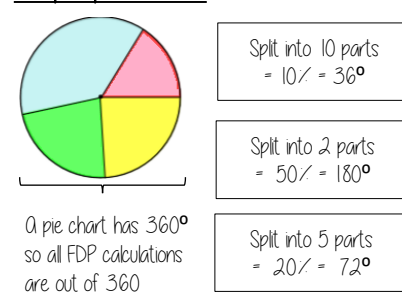
Percentages on a hundred grid



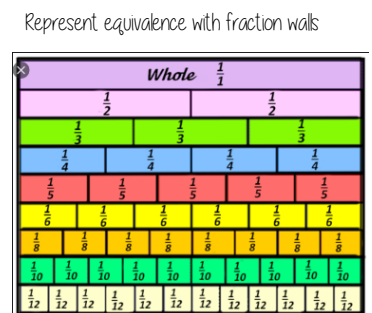
Quarters



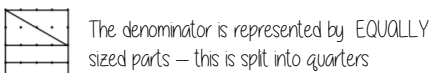
Simple pie charts



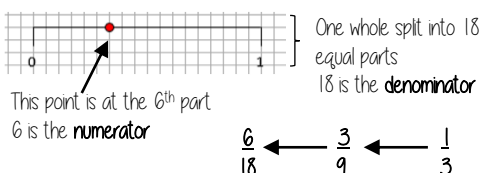
Equivalent fractions



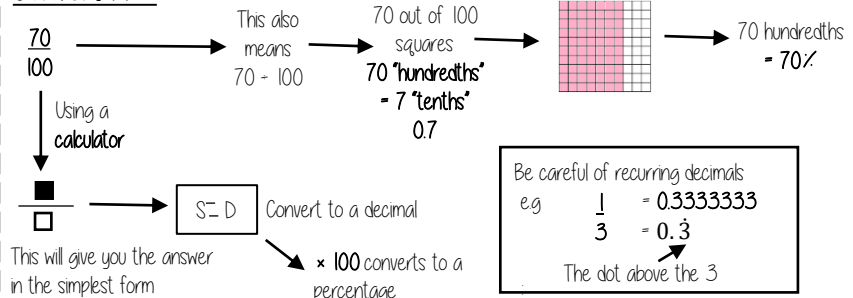
Fractions — on a diagram



Fractions — on a number line



Convert FDP



YEAR 7 — APPLICATION OF NUMBER

Solving problems with addition and subtraction

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand properties of addition/ subtraction
- Use mental strategies for addition/subtraction
- Use formal methods of addition/subtraction for integers
- Use formal methods of addition/subtraction for decimals
- Solve problems in context of perimeter
- Solve problems with finance, tables and timetables
- Solve problems with frequency trees
- Solve problems with bar charts and line charts

Keywords

Commutative: changing the order of the operations does not change the result

Associative: when you add or multiply you can do so regardless of how the numbers are grouped

Inverse: the operation that undoes what was done by the previous operation (The opposite operation)

Placeholder: a number that occupies a position to give value

Perimeter: the distance/ length around a 2D object

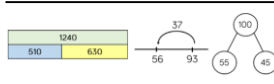
Polygon: a 2D shape made with straight lines

Balance: in financial questions — the amount of money in a bank account

Credit: money that goes into a bank account

Debit: money that leaves a bank account


Addition/ Subtraction with integers



Modelling methods for addition/ subtraction

- Bar models
- Number lines
- Part/ Whole diagrams

Addition is commutative



$$6 + 3 = 3 + 6$$

The order of addition does not change the result

Subtraction the order has to stay the same

$$360 - 147 = 360 - 100 - 40 - 7$$

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction
- Show your relationships by writing fact families

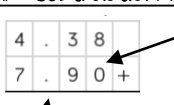
Formal written methods

	H	T	O
	1	8	7
+	5	4	2

	H	T	O
		4	2
-		2	4
			9


Remember the place value of each column
You may need to move 10 ones to the ones column to be able to subtract

Addition/ Subtraction with decimals



0 can be used to fill empty places with value

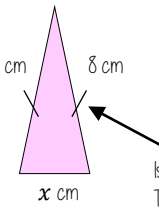
The decimal place acts as the placeholder and aligns the other values

If  represents 1 instead of 100

Revisit Fraction — Decimal equivalence
 $5.43 + 0.8$

Solve problems with perimeter

Perimeter is the length around the outside of a polygon



Isosceles Triangle notation

The triangle has a perimeter of 25cm
Find the length of x

$$8\text{cm} + 8\text{cm} + x\text{cm} = 25\text{cm}$$

$$16\text{cm} + x\text{cm} = 25\text{cm}$$

$$x\text{cm} = 9\text{cm}$$

Solve problems with finance

Profit = Income - Costs

Credit — Money coming into an account

Debit — Money leaving an account

Money uses a two decimal place system
14.2 on a calculator represents £14.20

Check the units of currency — work in the same unit

Tables and timetables

Distance tables

London	Cardiff	Glasgow
211	493	
556		
518	392	177

Belfast

This shows the distance between Glasgow and London
It is where their row and column intersects

Bus/ Train timetables

Harton	1005	1045	1130
Bridge	1024	1106	1147
Aville	1051	1133	1205
Ware	1117	1202	1233

Each column represents a journey, each row represents the time the 'bus' arrives at that location

TIME CALCULATIONS — use a number line

Two-way tables

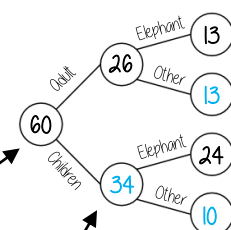
	H	T
H	HH	HT
T	TH	TT

Where rows and columns intersect is the outcome of that action

Frequency trees

60 people visited the zoo one Saturday morning

26 of them were adults. 13 of the adult's favourite animal was an elephant. 24 of the children's favourite animal was an elephant

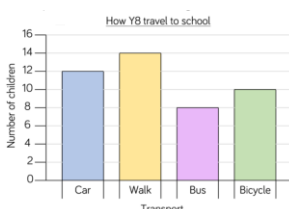


The overall total "60 people"

A frequency tree is made up from part-whole models
One piece of information leads to another

Probabilities or statements can be taken from the completed trees
e.g. 34 children visited the zoo

Bar and line charts



Use addition/ subtraction methods to extract information from bar charts

e.g. Difference between the number of students who walked and took the bus
Walk frequency — bus frequency

When describing changes or making predictions

- Extract information from your data source
- Make comparisons of difference or sum of values
- Put into the context of the scenario

YEAR 7 — APPLICATION OF NUMBER

Solving problems with multiplication and division

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and use factors
- Understand and use multiples
- Multiply/ Divide integers and decimals by powers of 10
- Use formal methods to multiply
- Use formal methods to divide
- Understand and use order of operations
- Solve area problems
- Solve problems using the mean

Keywords

Array: an arrangement of items to represent concepts in rows or columns

Multiples: found by multiplying any number by positive integers

Factor: integers that multiply together to get another number.

Mil: prefix meaning one thousandth

Centi: prefix meaning one hundredth

Kilo: prefix meaning multiply by 1000

Quotient: the result of a division

Dividend: the number being divided

Divisor: the number we divide by

Factors

Arrays can help represent factors

Factors of 10: 1, 2, 5, 10

Factors of 4: 1, 2, 4

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

The number itself is always a factor

Be strategic - Lay factors out in pairs can help you not to miss any

Square numbers have an ODD number of factors

Multiples

Bar models can represent by something is a multiple. Eg 20 is a multiple of 4

Lowest Common Multiples

LCM of 9 and 12: 36

The first time their multiples match

LCM = 36

Multiply/ Divide by powers of 10

$3 \times 100 = 300$

$0.03 \times 100 = 3$

Repeated multiplication and division by powers of 10 is commutative

$\div 10$ then $\div 10 \rightarrow \div 100$

Metric conversions

Useful Conversions

mm $\xrightarrow{\times 10}$ cm $\xrightarrow{\times 100}$ m $\xrightarrow{\times 1000}$ km

g $\xrightarrow{\times 1000}$ kg

ml $\xrightarrow{\times 1000}$ L

Division methods

Short division: $3584 \div 7 = 512$

Complex division: $\div 24 = \div 6 \div 4$

Break up the divisor using factors

Division with decimals: $24 \div 0.02 \rightarrow 24 \div 0.2 \rightarrow 240 \div 2$

All give the same solution as represent the same proportion

Multiply the values in proportion until the divisor becomes an integer

Multiplication methods

Long multiplication (column)

Grid method

Repeated addition

Multiplication with decimals

Perform multiplications as integers e.g. $0.2 \times 0.3 \rightarrow 2 \times 3$

Make adjustments to your answer to match the question: $0.2 \times 10 = 2$, $0.3 \times 10 = 3$

Therefore $6 \div 100 = 0.06$

Order of operations

Brackets

Indices or roots

Multiplication or division

Addition or subtraction

If you have multiple operations from the same tier work from left to right

e.g. $10 - 3 + 5 \rightarrow 10 - 3 \rightarrow 7 + 5$

$6 \times 4 + 8 \times 2 = 24 + 16 = 40$

Area problems

Rectangle: Base x Perpendicular height

Parallelogram/ Rhombus: Base x Perpendicular height

Triangle: $\frac{1}{2} \times$ Base x Perpendicular height

A triangle is half the size of the rectangle it would fit in

Mean problems

Mean — a measure of average. It gives an idea of the central value.

Lilly, Annie and Ezra have the following cubes

Finding the mean amount is the average amount each person would have if shared out equally

The mean number of blocks would be 8 each

YEAR 7 — APPLICATION OF NUMBER

Fractions and percentages of amounts

What do I need to be able to do?

- By the end of this unit you should be able to:
- Find a fraction of a given amount
 - Use a given fraction to find the whole or other fractions
 - Find the percentage of an amount using mental methods
 - Find the percentage of a given amount using a calculator

Keywords

- Fraction:** how many parts of a whole we have
Equivalent: of equal value
Whole: a number with no fractional or decimal part
Percentage: parts per 100 (uses the % symbol)
Place Value: the value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right
Convert: change into an equivalent representation, often fraction to decimal to a percentage cycle.

Fraction of a given amount

Find $\frac{2}{5}$ of £205

The bar represents the whole amount

£205

£41

2 out of the 5 equal parts
 $2 \times £41 = \underline{£82}$

$£205 \div 5 = £41$

Each part of the bar model represents £41

90

30 30 30

15 15 15

Use bar models for comparisons

$\frac{1}{3}$ of 90 = 30

$\frac{2}{3}$ of 45 = 30

$\therefore \frac{1}{3}$ of 90 = $\frac{2}{3}$ of 45

Use a fraction of amount

$\frac{2}{3}$ of a value is 70. What is the whole number?

$70 \div 2 = 35$

Each part of the bar model represents 35

70

35 35 35

$35 \times 3 = 105$

The whole number is 105

The wording of the question is important to setting up the bar model

$\frac{3}{4}$ of a number is 63.

Find the whole

63

21 21 21 21

What is $\frac{1}{6}$ of the number?

Use the whole to find a given part

84

14 14 14 14 14 14

= 14

Find the percentage of an amount (Mental methods)

The whole represents 100%

10% = $\frac{1}{10}$ of the whole

$10\% = \frac{1}{10}$ of the whole $50\% = \frac{5}{10} = \frac{1}{2}$ of the whole

$20\% = \frac{2}{10} = \frac{1}{5}$ of the whole $5\% = \frac{1}{20}$ of the whole

Find 65% of 80

80

8 8 8 8 8 8 8 8 8 8

Method 1
 $65\% = 10\% \times 6 + 5\%$
 $= (8 \times 6) + 4$
 $= 52$

Method 2
 $65\% = 50\% + 10\% + 5\%$
 $= 40 + 8 + 4$
 $= 52$

For bigger percentages it is sometimes easier to take away from 100%

Find the percentage of an amount (Calculator methods)

Using a multiplier

Find 65% of 80

Fraction, decimal, percentage conversion

$65\% = \frac{65}{100} = 0.65$ ← The multiplier

$0.65 \times 80 = 52$

Using the percent button

Find 65% of 80

Type 65

Press **SHIFT** **C** **(%)**

Press **×** 80 and then press =

This brings up the % button on screen
 You will see 65%

You can also use the calculator to support non calculator methods and find 1% or 10% then add percentages together

"of" can represent 'x' in calculator methods

YEAR 7 — DIRECTED NUMBER

Operations with equations and directed numbers

What do I need to be able to do?

- By the end of this unit you should be able to:
- Perform calculations that cross zero
 - Add/ Subtract directed numbers
 - Multiply/ Divide directed numbers
 - Evaluate algebraic expressions
 - Solve two-step equations
 - Use order of operations with directed number

Keywords

- Subtract:** taking away one number from another.
Negative: a value less than zero.
Commutative: changing the order of the operations does not change the result.
Product: multiply terms.
Inverse: the opposite function.
Square root: a square root of a number is a number when multiplied by itself gives the value (symbol $\sqrt{\quad}$)
Square: a term multiplied by itself.
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

Perform calculations that cross zero

Number lines are useful to help you visualise the calculation crossing 0

$4 - 6 = -2$

Use the number line to guide subtraction of 6

Start at 4

Find the difference between 6 and -4

From 6 to 0
6
From 0 to -4
4
10 beads between them

$-5 + 5 = 0$ Rearrangements of the same equation $5 - 5 = 0$

Add directed numbers

$2 + -4 = -2$

Zero pair $(-1 + 1 = 0)$

Two -1 's left $= -2$

$8 + -3 = 5$

Partitioning

$8 + -3 = 5$ $5 + 3 + -3 = 5$

Partition the value to create a zero pair calculation

Generalisation $+ - = -$

Subtract directed numbers

Representation for calculation

$2 - -1 = 3$

Take away one

Start with the representation of 2

$2 - -3 = 5$

Generalisation $- - = +$

Multiply/ Divide directed numbers

Two representations of the same calculation $2 \times -3 = -6$

Negative, Negative calculation

-2×-3

This is the negative of 2×-3

$-2 \times -3 = 6$

The act of making counters into their negative is turning them over

Divisions are the inverse operations

Evaluate algebraic expressions

$a = 5$ $b = -4$

$a^2 = 5^2$ $b^2 = (-4)^2$
 $a^2 = 25$ $b^2 = 16$

With negative numbers the brackets are important so that it performs -4×-4 .

Brackets around negative substitutions helps remove calculation errors

$2a - b = 2 \times 5 - (-4) = 10 + 4 = 14$

$3b - 2a = 3(-4) - 2(5) = -12 - 10 = -22$

Two-step equations

Bar Model

$4x + 2 = 10$

Representing the same question (use fact families)

$10 - 4x = 2$

Function machine

$x \rightarrow \times 4 \rightarrow +2 \rightarrow 10$

Inverse operations to find x

Use order of operations

Brackets

Indices or roots

Multiplication or division

Addition or subtraction

Remember square roots have a positive and negative value

x	-3	-2	-1	0	1	2	3
-3	9	6	3	0	-3	-6	-9
-2	6	4	2	0	-2	-4	-6
-1	3	2	1	0	-1	-2	-3
0	0	0	0	0	0	0	0
1	-3	-2	-1	0	1	2	3
2	-6	-4	-2	0	2	4	6
3	-9	-6	-3	0	3	6	9